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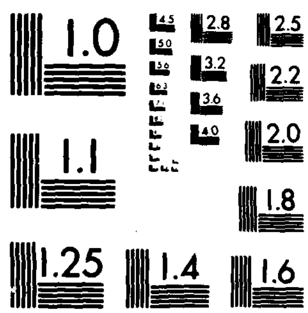

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EXPLOSION DRIVEN MAGNETOGASDYNAMIC FLOWS WITH HIGH MAGNETIC REYNOLDS AND INTERACTION NUMBERS

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1 December 1981

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CONTENTS

INTRODUCTION

This Annual Progress Report contains investigations on detonation driven magnetogasdynamic and solid state power generators and related research problems. The work was conducted under ONR Contract N00014-81-WR-10107 in the period from February 1 to September 30, 1981.

The electric currents and voltages produced by electromagnetic induction in explosion driven magnetogasdynamic generators with homogeneous external magnetic fields $\vec{B}_o$ are calculated for two shock flow models, (1) the (implosion produced) jet flow $v(t) = v_oH(t)$ with constant shock speed v_o, and (2) the plane detonation flow with shock speed $v(t) \propto t^{-1/3}$ (similarity solution). External load circuits with resistance R_o and inductance L_o are considered which are connected i) to the upstream and ii) to the downstream ends of the generator electrodes so that the magnetic self-fields $\tilde{\vec{B}}$ of the electric currents are i) antiparallel and ii) parallel to the external [illegible]ic field $\vec{B}_o$. It is shown that optimum power is generated for generators with jet shock flow and load circuits of type (ii), i.e., positive superposition of induced and external magnetic fields. In this case, electric energies $\Delta E \sim 10^5$ Joule can be generated with external fields $B_o = 1$ Tesla per detonation. The importance of the results in the

Chapters II and III for the design of detonation driven generators is obvious.

One of the main technical problems of explosion driven magnetogasdynamic generators is the requirement of large external magnetic fields $B_o > 1$ Tesla, which are difficult to procure in connection with military applications, For this reason, a solid state power generator is analyzed in which the driving electromotoric force is due to electric polarization in the stress field of an explosion produced shock wave. The solid is assumed to be a one-dimensional slab between plane electrodes, which are connected through an external load circuit. The temporal current build-up in the external circuit, due to electric polarization and electric conduction behind the shock front of the stress wave, space charge and electric polarization relaxations, is determined by an inhomogeneous Volterra integral equation. For shocked solids with cross section of the order of one square meter, energies of the order $\Delta E \sim 10^5$ Joule per pulse can be achieved. This scheme for an electric generator, which does not require external magnetic fields, appears to be promising for experimental investigation (Chapter IV).

An other method of avoiding large external magnetic fields $B_o \geq 1$ Tesla is to build an explosion driven MGD generator, in which a weak external magnetic field $B_o \sim 10^{-2}$ Tesla is compressed between the plasma shock front (high magnetic Reynolds numbers $R_m = \mu_o \sigma va$) and a copper liner. The exact relativistic theory of the flux compression MGD generator leads to two coupled integrodifferential equations which appear to be unsolvable by analytical methods. For this reason, the electromagnetic diffusion and induction in accelerated conductors in external $\vec{B}_o$ fields was analyzed in the parabolic diffusion approximation, which is valid for small relaxation times $\tau = \varepsilon_o/\sigma$ ($\tau \sim 10^{-18}$ sec for copper, $\sigma = 6\times10^7\Omega^{-1}/m$). The conventional boundary conditions (which violate the continuity of the tangential electric field) were replaced by new boundary conditions, which consider the electromagnetic transients outside of the conductor. Thus, a realistic evaluation of magnetic flux compression and magnetic flux losses through the plasma piston and the copper liner is accomplished (Chapters V and VI).

In Chapter VII, a quantum-kinetic theory of the anomalous plasma heating by high-frequency electromagnetic fields is presented based on the Maxwell-Vlasov and Schroedinger equations. This investigation has been carried through in collaboration with visiting Prof. S.H. Kim.

The work on flux compression electric power generators will be communicated in the 1982 Annual Report. There, also new types of explosion driven generators will be discussed, which do not require an external magnetic field.

DETONATION DRIVEN INDUCTION GENERATOR

WITH ANTIPARALLEL EXTERNAL AND INDUCED MAGNETIC FIELDS

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Abstract

Based on Maxwell's equations, the electric current and voltage pulses induced in explosion driven induction generators ($R_m = \mu_o \sigma v L >> 1$) with plane electrodes and homogeneous external magnetic field $\vec{B}_o$ are calculated analytically for two plasma shock flow models, (i) the jet flow with shock speed $v(t) = v_o H(t)$ and (ii) the plane detonation flow with shock speed $v(t) = (2/3)(E_o/\rho_o)^{1/3} t^{-1/3}$. The external load circuit with resistance R_o and inductance L_o is connected to the flow entrance ends of the electrodes so that the magnetic self-field $\vec{B}(t)$ of the generated current $I(t)$ is antiparallel to the transverse external field $\vec{B}_o$. It is shown that the jet flow produces large current and voltage pulses with extended plateaus, whereas the detonation flow generates considerably smaller and shorter current and voltage pulses. In both cases, the magnetic self-field $\vec{B}(t)$ of the generated current $I(t)$ is of the order-of-magnitude of $\vec{B}_o$, and its negative superposition on the external magnetic field is not advantageous for power generation.

INTRODUCTION

In contrast to the conductive magnetohydrodynamic generator with steady-state magnetic and flow fields [1], induction generators are based on the electromagnetic interaction of a conducting flow field with a magnetic field, where at least one of the fields is transient [2] so that voltages are induced due to temporal flux changes $d\Phi/dt \neq 0$. In some induction generators, a detonation plasma with a decreasing shock velocity $v(t) \to 0$ for $t \to \infty$ is directly injected into the inter-electrode space [3], whereas in more recent experiments[4,5] discontinuous plasma jets are shot across an external, constant magnetic field. The plasma jets $v(t) \doteq v_o H(t)$ of nearly constant shock speed v_o are produced, e.g., by implosion of a dense noble gas between flyer plates and subsequent expansion through a diaphragm into the generator channel [4], or by means of a tube like arrangement of the solid explosive resulting in a cylindrical implosion with large axial shock velocity of the filling gas [5].

Detonation driven induction generators have shock velocities $v = 10^4 - 10^5$ m/sec, external magnetic fields $B_o = 1 - 10$ Tesla, conductivities $\sigma > 10^4\ \Omega^{-1}$/m, and flow energies $\frac{1}{2}\rho v^2 \sim 10^{10} - 10^{12}$ Joule m^{-3} at pressures $p > 10^4$ bar ($\rho > 10^2$ kg m^{-3}). The corresponding magnetic Reynolds number R_m is large, whereas the magnetic interaction number R_i is small,

$$R_m = \mu_o \sigma v L \gg 1 \quad , \quad R_i = B_o^2/\mu_o \rho v^2 \ll 1 \quad , \tag{1}$$

where $L \sim |\nabla \times \vec{B}/B|$. For $R_m \gg 1$, the electric current is restricted to a thin layer $\delta \sim (\hat{t}/\mu_o\sigma)^{1/2}$ behind the shock front ($\hat{t}$ = transit time of flow in $\vec{B}_o$). In view of the large dynamic pressure $\frac{1}{2}\rho v^2$ compared to the magnetic pressure $B_o^2/2\mu_o = 10^6 - 10^8$ Joule m^{-3}, the efficiency of energy transformation is small, while the performance is significant since currents $I \sim (B_o/\mu_o)b > 10^4$ Amps (b = electrode width) and energies $\overline{UI\hat{t}} > 10^4$ Joule have been produced experimentally per detonation [4,5].

Herein, the initial-value problem for a detonation driven generator is solved in closed form by means of Maxwell's equations, in which the flow entrance ends of the coplanar electrodes are connected by an external L_o-R_o load circuit (Fig. 1) so that the magnetic field $\vec{B}(t)$ of the generated currents I(t) is antiparallel to the external magnetic field $\vec{B}_o$. Two types of plasma shock flows are considered corresponding to i) the idealized jet flow with constant shock speed v_o and ii) the plane detonation flow due to an explosive energy release E_o per unit area in a gas of initial density ρ_o:

$$v(t) = v_o H(t) \quad , \qquad \hat{x}(t) = v_o t H(t) \quad ,$$
$$0 < t < \hat{t} \quad , \qquad \hat{t} = x_\infty / v_o \quad ; \tag{2}$$

and

$$v(t) = (2/3)(E_o/\rho_o)^{1/3} t^{-1/3} \quad , \quad \hat{x}(t) = (E_o/\rho_o)^{1/3} t^{2/3} \quad ,$$
$$0 < t < \hat{t} \quad , \quad \hat{t} = (E_o/\rho_o)^{-1/2} x_\infty^{3/2} \quad ; \tag{3}$$

where $\hat{x}(t)$ is the shock front position, and $\hat{t}$ is the transit time of the shock front $\hat{x}(t)$ in the duct of effective length x_∞ (Fig. 1). The self-similar solution (3) is asymptotically correct for larger times t [6], but can be used for t > 0 since the singularity of v(t) at t = 0 is without physical consequences for generator applications.

The external circuit is assumed to be located at $x = -x_o$ (Fig. 1), where x_o is an effective distance which can be calculated once the geometries of the inductance L_o and resistance R_o are specified.

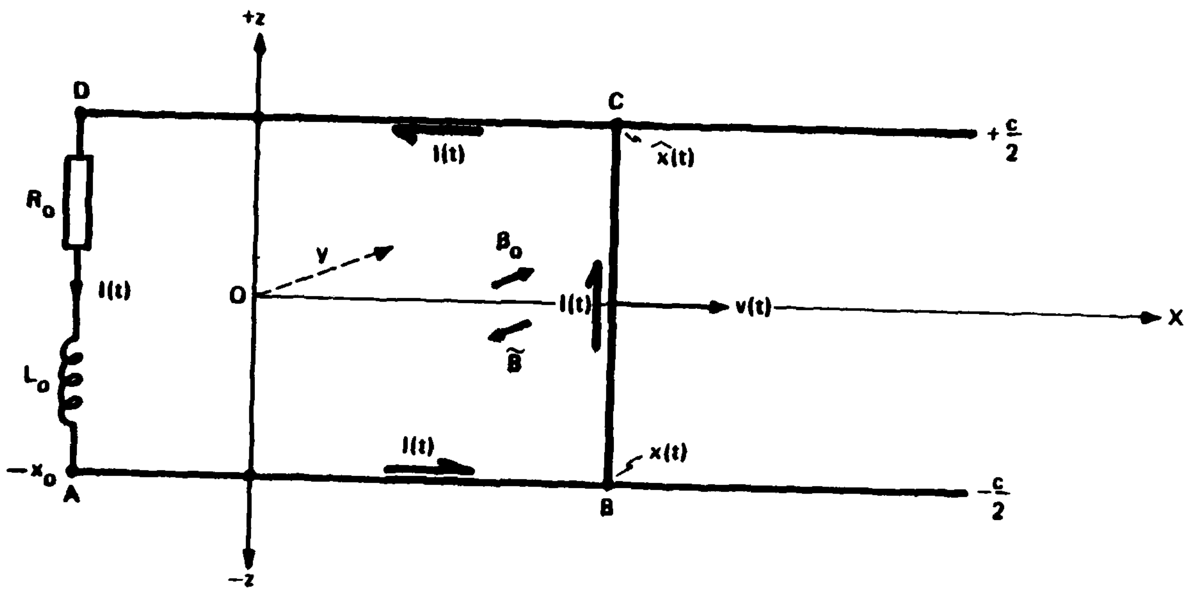

FIG. 1: Generator model with plane electrodes $z = \pm c/2$, applied magnetic field $\vec{B}_o$, and $R_o - L_o$ load circuit AD. Shock front BC at $x = \hat{x}(t) > 0$.

INITIAL-VALUE PROBLEM

Fig. 1 exhibits a one-dimensional model for a detonation driven generator, in which the shock wave flow $v(t) = d\hat{x}(t)/dt$ is initiated at time $t = 0$ in the plane $x = 0$ by explosive charges in the space $-x_o < x < 0$ so that the shock front is at $\hat{x}(t) \geq 0$ for $t \geq 0$. The Cu-electrodes in the planes $z = \pm c/2$ have the extensions $-x_o < x < x_\infty < \infty$ and $-b/2 < y < +b/2$, and are connected by an external circuit AD with load R_o and inductance L_o. As a result of the $\vec{v}\times\vec{B}_o$ interaction of the conducting shock front ($\sigma > 10^4 \Omega^{-1}/m$) with the external magnetic field $\vec{B}_o = \{0,B_o,0\}$, a current $I(t)$ is induced behind the shock front BC in a cross section $\Delta x \Delta y = b\delta$ where $\delta \sim (\hat{t}/\mu_o\sigma)^{1/2}$ is the (small) width of the current layer. The current $I(t)$ flows in the closed loop $A \to B \to C \to D \to A$, where the leg BC at $\hat{x}(t)$ moves with the shock speed $v(t) = d\hat{x}(t)/dt$.

Application of the surface curl equation $\vec{n}\times[\vec{B}] = \mu_o \vec{I}(t)/b$ to the interfaces AB, BC, CD, DA demonstrates - in the one-dimensional approximation neglecting end effects at $y = \pm b/2$ and $x = -x_o$ - that the current $I(t)$ produces a magnetic self-field in the extending loop volume,

$$\vec{B}(t) = \{0,-\mu_o I(t)/b,0\} \ , \quad -x_o < x < \hat{x}(t) \ , \quad |y| < \frac{b}{2}, \ |z| < \frac{c}{2} \qquad , \quad (4)$$

which is antiparallel to the external magnetic field $\vec{B}_o$. The total magnetic field within the loop volume is

$$\vec{B}(t) = \{0,B_o-\mu_o I(t)/b,0\} \quad , \quad -x_o < x < \hat{x}(t) \ , \quad |y| < \frac{b}{2} \ , \quad |z| < \frac{c}{2} \qquad , \quad (5)$$

whereas $\vec{B}(t) = \vec{B}_o$ outside of this region. Integration of the Maxwell equation $\nabla\times\vec{E} = -\partial\vec{B}/\partial t$ over the cross secti ABCD of ıe extending current loop yields by Stokes' theorem

$$\oint \vec{E}^* \cdot d\vec{r} = - \oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint (\vec{v}\times\vec{B}) \cdot d\vec{r} \qquad (6)$$

where

$$\vec{E}^* = \vec{E} + \vec{v}\times\vec{B} \quad , \quad v^2 << 1/\mu_o\varepsilon_o \quad , \tag{7}$$

and $\vec{v}(t)$ is the velocity field of the sections of the closed loop, at the point corresponding to the path element $d\vec{r}$ [$\vec{v}(t) = \vec{e}_x d\hat{x}(t)/dt$ for BC, but $\vec{v}(t) = \vec{0}$ for CD, DA, AB]. The surface element $d\vec{S} = -\vec{e}_y cdx$ forms with $d\vec{r}$ [pointing everywhere in the direction $\vec{I}(t)$] a right hand system. By Eqs. (6) and (7)

$$(R_o + R_p)I(t) + L_o\frac{dI(t)}{dt} = -c\int_{-x_o}^{\hat{x}(t)} \frac{d\tilde{B}(t)}{dt}dx + \int_{-c/2}^{+c/2} \frac{d\hat{x}(t)}{dt}[B_o + \tilde{B}(t)]dz \tag{8}$$

since $\int_B^C E^* dz = IR_p$ (R_p = plasma resistance), $\int_D^A Edz = R_oI + L_o dI/dt$, and $\int Edz \approx 0$ for the sections AB and CD of the highly conducting electrodes ($\sigma_{Cu} \sim 10^8\Omega^{-1}/m$). Accordingly,

$$(R_o + R_p)I(t) + \frac{d}{dt}\{[L_o + \tilde{L}(t)]I(t)\} = cB_o\frac{d\hat{x}(t)}{dt} \quad , \tag{9}$$

where

$$\tilde{L}(t) = \mu_o c[x_o + \hat{x}(t)]/b > 0 \quad , \quad 0 \leq \hat{x}(t) \leq x_\infty \quad , \tag{10}$$

is the time-dependent inductance of the electrode sections AB & DC of increasing length $\Delta x = x_o + \hat{x}(t)$. Multiplication of Eq. (9) by I(t) leads to the conservation law

$$(R_o + R_p)I^2 + \frac{d}{dt}\{\frac{1}{2}[L_o + \tilde{L}(t)]I^2\} + \frac{\mu_o c}{2b}I^2\frac{d\hat{x}(t)}{dt} = I\frac{d\hat{x}(t)}{dt}B_o c \tag{11}$$

according to which the ohmic power dissipation, the temporal change of the magnetic energy, and the work on the current sheath per unit time equal the power input $\hat{U}I$.

The plasma resistance $R_P = R_P(t)$ is a slowly varying function of t and can be calculated only via a detailed physical model of the current sheath δ at $x \le \hat{x}(t)$. However, since R_o is sufficiently large in actual experiments, $R_o + R_P$ can be treated as a quasi-constant,

$$R_o + R_P(t) \doteq \text{constant}, \quad R_o > R_P \; ; \qquad R_o + R_P(t) = R_o \; , \quad R_o >> R_P \; . \tag{12}$$

For a generalized formulation, dimensionless dependent and independent variables are introduced by

$$J(\tau) = I(t)/I_o \; , \quad \tau = t/t_o \; , \quad \hat{\xi}(\tau) = \hat{x}(t)/c \tag{13}$$

with

$$I_o = (B_o/\mu_o)b \; , \quad t_o = (\mu_o c^2/b)/(R_o + R_P) \; , \quad \Lambda_o = (x_o/c) + L_o/(\mu_o c^2/b). \tag{14}$$

By Eqs. (9) - (10) and (13) - (14), the electric current I(t) in the explosion driven generator is determined by the inhomogeneous initial-value problem with variable coefficients:

$$\frac{d}{d\tau}\{[\Lambda_o + \hat{\xi}(\tau)]J(\tau)\} + J(\tau) = \frac{d\hat{\xi}(\tau)}{d\tau} \; , \quad 0 < \tau < \hat{\tau} \; , \tag{15}$$

$$J(\tau = 0) = 0 \; . \tag{16}$$

Eqs. (15) - (16) are solved for the jet and detonation flows in Eqs. (2) and (3), respectively, which define the dimensionless coordinate $\hat{\xi}(\tau)$ of the shock front.

In view of the large flow energy, $\frac{1}{2}\rho v^2 >> B_o^2/2\mu_o$, it is not necessary to correct the above generator model for velocity decreases due to Lorentz and viscous forces. Furthermore, viscous boundary layers are disregarded since the plasma moves in form of a rectangular velocity slug through the duct in large Reynolds number flow, $R = \rho v L/\mu >> 1$.

JET-FLOW SOLUTION

For the jet flow, the dimensionless shock front coordinate $\hat{\xi}(\tau)$, ξ_o, and the transit time $\hat{\tau}$ are by Eqs. (2) and (13)

$$\hat{\xi}(\tau) = \xi_o\tau \quad , \quad \xi_o = \frac{v_o}{c}\left(\frac{\mu_o c^2/b}{R_o+R_P}\right) \quad , \quad \hat{\tau} = \frac{x_\infty}{v_o}\left(\frac{R_o+R_P}{\mu_o c^2/b}\right) \quad . \tag{17}$$

Since $\hat{\xi}(\tau) \geq 0$ for $\tau \geq 0$, the initial-value problem (15) - (16) is without singularities, and has the closed-form solution:

$$J(\tau) = \frac{\xi_o}{1+\xi_o}\left[1 - \left(1 + \frac{\xi_o}{\Lambda_o}\tau\right)^{-(1+\xi_o)/\xi_o}\right] \quad , \quad 0 \leq \tau \leq \hat{\tau} \quad . \tag{18}$$

The transit time in Eq. (17) satisfies the inequality,

$$(\xi_o/\Lambda_o)\hat{\tau} = \frac{x_\infty}{c}/\left[\frac{x_o}{c} + \frac{L_o}{\mu_o c^2/b}\right] >> 1 \ , \quad \text{for } x_\infty >> x_o \ , \quad L_o \leq (x_o/c)\mu_o c^2/b \ , \tag{19}$$

which indicates that the independent variable $(\xi_o/\Lambda_o)\tau$ may assume large values. Eq. (18) has the limits:

$$J(\tau) = (\xi_o/\Lambda_o)\tau \quad , \quad (\xi_o/\Lambda_o)\tau << 1 \quad , \tag{20}$$

$$J(\tau) = \xi_o/(1 + \xi_o) \quad , \quad (\xi_o/\Lambda_o)\tau >> 1 \quad . \tag{21}$$

Accordingly, the maximum current is reached only asymptotically for large times, since $J(\tau)$ is a monotonically increasing function of $0 \leq \tau \leq \hat{\tau}$,

$$J_{max} = \xi_o/(1 + \xi_o) \quad , \quad I_{max} = \frac{B_o}{\mu_o} b\xi_o/(1 + \xi_o), \quad (\xi_o/\Lambda_o)\tau >> 1 \quad . \tag{22}$$

The generated voltage, $U = R_o I + L_o dI/dt$, is by Eq. (18) in dimensionless representation

$$\mathcal{U}(\tau) = \frac{\xi_o}{1+\xi_o}[1 - (1+\frac{\xi_o}{\Lambda_o}\tau)^{-(1+\xi_o)/\xi_o}] + \frac{L_o/R_o}{t_o}\frac{\xi_o}{\Lambda_o}(1+\frac{\xi_o}{\Lambda_o}\tau)^{-(1+2\xi_o)/\xi_o} \tag{23}$$

where $U_o = I_o R_o$. Eq. (23) has the asymptotic values

$$\mathcal{U}(\tau) = [1 - \frac{L_o/R_o}{t_o}(\frac{1+2\xi_o}{\Lambda_o})]\frac{\xi_o}{\Lambda_o}\tau + \frac{L_o/R_o}{t_o}\frac{\xi_o}{\Lambda_o} \quad , \quad \frac{\xi_o}{\Lambda_o}\tau \ll 1 \quad , \tag{24}$$

$$\mathcal{U}(\tau) = \frac{\xi_o}{1+\xi_o} \quad , \quad \frac{\xi_o}{\Lambda_o}\tau \gg 1 \quad . \tag{25}$$

For $1 + (\xi_o/\Lambda_o)\tau = (L_o/R_o t_o)(1 + 2\xi_o)/\Lambda_o$, $\mathcal{U}(\tau)$ assumes a minimum,

$$\mathcal{U}_{min} = \frac{\xi_o}{1+\xi_o}\{1 - \frac{\xi_o}{1+2\xi_o}[\frac{L_o/R_o}{t_o}(\frac{1+2\xi_o}{\Lambda_o})]^{-(1+\xi_o)/\xi_o} \quad . \tag{26}$$

Accordingly, the maximum voltage value is reached only asymptotically for large times, $(\xi_o/\Lambda_o)\tau \gg 1$,

$$\mathcal{U}_{max} = \xi_o/(1+\xi_o) \quad , \quad U_{max} = I_o R_o \xi_o/(1+\xi_o) \quad . \tag{27}$$

<u>Example</u>. For $b = 10^{-1}$m, $c = 1$ m, $x_o = 10^{-1}$m, $x_\infty = 1$ m, $\sigma = 10^5 \Omega^{-1}$/m, $R_p \simeq \sigma^{-1}c/b\delta = 10^{-2}\Omega$, $R_o + R_p = 2\times10^{-1}\Omega$, $L_o = 4\pi\times10^{-6}$Henry, $v_o = 10^4$m/sec, $B_o = 1$ Tesla, the characteristics of the generator are: $t_o = 2\pi\times10^{-5}$sec, $\hat{t} = 10^{-4}$sec, $\xi_o = 2\pi/10$, $\Lambda_o = 11/10$, $I_o = 10^6/4\pi$ Amps, $I_{max} \simeq 3\times10^4$ Amps, $U_{max} \simeq 6\times10^3$Volt, $(UI)_{max} \simeq 2\times10^8$ Watt, $\Delta E = \overline{UI\hat{t}} \sim 10^4$ Joule.

Fig. 2 shows the current $J(\tau)$ versus $(\xi_o/\Lambda_o)\tau$ for $\xi_o = 2\pi/10$ and $\xi_o = 9/10$ based on Eq. (19). The distribution $J(\tau)$ rises steeply within $0 < (\xi_o/\Lambda_o)\tau < 1$ and then flattens out into a shightly increasing "plateau" for larger times τ. In experiments [4,5], the "plateau" is more constant or even decreases somewhat with increasing time t, which appears to be due to i) shock speeds v(t) decreasing slightly with time t, and ii) external circuit leads to the center of the electrodes.

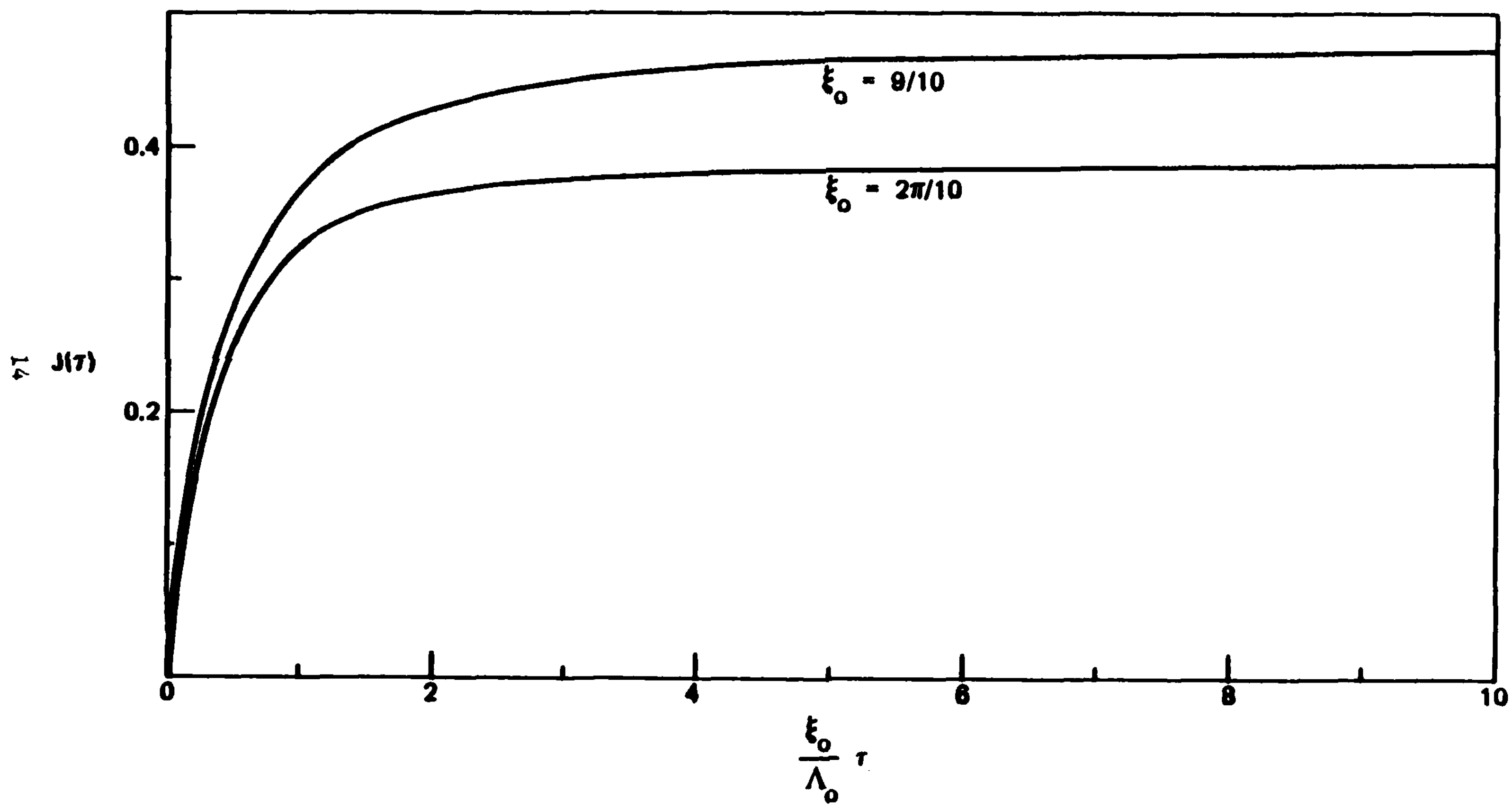

FIG. 2: Current transient $J(\tau)$ of jet shock flow versus $(\xi_o/\Lambda_o)\tau$ for $\xi_o = 2\pi/10;\ 9/10$.

DETONATION-FLOW SOLUTION

The dimensionless shock front coordinate $\hat{\xi}(\tau)$, ξ_o, and the transit time $\hat{\tau}$ for the detonation flow follow from Eqs. (3) and (13) as

$$\hat{\xi}(\tau) = \xi_o \tau^{2/3} \quad , \quad \xi_o = \left(\frac{E_o}{\rho_o}\right)^{1/3} c^{-1} \left(\frac{\mu_o c^2/b}{R_o + R_p}\right)^{2/3} \quad , \quad \hat{\tau} = \xi_o^{-3/2} \left(\frac{x_\infty}{c}\right)^{3/2} . \tag{28}$$

Since $\hat{\xi}(\tau) > 0$ for $\tau > 0$, the solution of the initial-value problem (15) - (16) is representable in terms of an indefinite integral with regular integrand:

$$J(\tau) = 1 - \frac{1 + \Lambda_o^{-1} \int_0^\tau e^{3\xi_o^{-1}[\tau^{1/3} - \alpha_o^{-1} \arctan(\alpha_o \tau^{1/3})]} d\tau}{(1 + \alpha_o^2 \tau^{2/3}) e^{3\xi_o^{-1}[\tau^{1/3} - \alpha_o^{-1} \arctan(\alpha_o \tau^{1/3})]}} \quad , \quad 0 < \tau < \hat{\tau} \quad , \tag{29}$$

where

$$\alpha_o \equiv (\xi_o / \Lambda_o)^{1/2} \quad , \tag{30}$$

Eq. (29) is an integral functional of $\alpha_o \tau^{1/3}$, where

$$\alpha_o \hat{\tau}^{1/3} = \left\{\frac{x_\infty}{c} / \left[\frac{x_o}{c} + \frac{L_o}{\mu_o c^2/b}\right]\right\}^{1/2} \gg 1 \text{ , for } x_\infty \gg x_o \text{ , } L_o \lesssim (x_o/c)\mu_o c^2/b. \tag{31}$$

For small and large times, Eq. (29) gives the limits

$$J(\tau) = \alpha_o^2 \tau^{2/3} \quad , \quad \alpha_o \tau^{1/3} \ll 1 \quad , \tag{32}$$

$$J(\tau) = 0 \quad , \quad \alpha_o \tau^{1/3} \gg 1 \quad . \tag{33}$$

$J(\tau)$ has a true maximum J_{max} at time $\tau = \bar{\tau}$ where $dJ(\tau)/d\tau = 0$ for $\tau = \bar{\tau}$. By Eqs. (15), (28), and (29), J_{max} and $\bar{\tau}$ are given by

$$J_{max} = \xi_o/(\xi_o + \frac{3}{2}\tilde{\tau}^{1/3}) \quad , \quad (34)$$

$$\frac{\tilde{\tau}^{1/3}}{\frac{2}{3}\xi_o + \tilde{\tau}^{1/3}} = \frac{1 + \Lambda_o^{-1}\int_0^{\tilde{\tau}} e^{3\xi_o^{-1}[\tau^{1/3} - \alpha_o^{-1}\arctan(\alpha_o\tau^{1/3})]} d\tau}{(1 + \alpha_o^2\tilde{\tau}^{2/3})e^{3\xi_o^{-1}[\tilde{\tau}^{1/3} - \alpha_o^{-1}\arctan(\alpha_o\tilde{\tau}^{1/3})]}} \quad . \quad (35)$$

The generated voltage, $U_o = R_oI + L_o dI/dt$, is by Eq. (29) in dimensionless representation

$$\mathcal{U}(\tau) = J(\tau) - \frac{L_o/R_o}{t_o}[(1 + \frac{2}{3}\xi_o\tau^{-1/3})J(\tau) - \frac{2}{3}\xi_o\tau^{-1/3}]/\Lambda_o(1 + \alpha_o^2\tau^{2/3}),\ 0 < \tau < \tilde{\tau}. \quad (36)$$

The inductive singularity, $\mathcal{U}(\tau) \to \infty$ for $\tau \to 0$, is due to the divergence of the selfsimilar speed [6], $d\hat{\xi}(\tau)/d\tau = (2\xi_o/3)\tau^{-1/3} \to \infty$ for $\tau \to 0$, and has, therefore, no physical meaning.

<u>Example</u>. For $b = 10^{-1}$m, $c = 1$ m, $x_o = 10^{-1}$m, $x_\infty = 1$ m, $\sigma = 10^5\Omega^{-1}$/m, $R_p \doteq \sigma^{-1}c/b\delta = 10^{-2}\Omega$, $R_o + R_p = 2\times10^{-1}\Omega$, $L_o = 4\pi\times10^{-6}$ Henry, $v(10^{-6}\text{sec}) = 2.650\times10^4$m/sec, $B_o = 1$Tesla, the generator characteristics are: $t_o = 2\pi\times10^{-5}$sec, $\hat{t} = 1.262\times10^{-4}$sec, $\xi_o = 2\pi/10$, $\Lambda_o = 11/10$, $I_o = 10^6/4\pi$ Amps, $I_{max} \doteq 1\times10^4$ Amps, $U_{max} \doteq 2\times10^3$ Volt, $(UI)_{max} \doteq 2\times10^7$ Watt, $\Delta E = \overline{UI}\hat{t} \sim 10^3$ Joule.

Fig. 3 exhibits the current $J(\tau)$ versus $\alpha_o\tau^{1/3}$ for $\xi_o = 2\pi/10$ with $\Lambda_o = 1/4\pi$, $10/4\pi$, and $100/4\pi$ as parameter based on Eq. (29). In each case, $J(\tau)$ rises rapidly to a maximum value and then decays slowly with increasing time. The peak current J_{max} decreases with increasing Λ_o (inductance). It should be noted that the distributions $J(\tau)$ are applicable up to the transit value $\alpha_o\hat{\tau}^{1/3} = (x_\infty/\Lambda_o c)^{1/2}$ [Eq. (28)] which depends on the respective system parameters. Experimental current oscillograms for the detonation flow[3] agree only qualitatively with Eq. (29) since the electrodes were connected in their centers by the external load circuit.

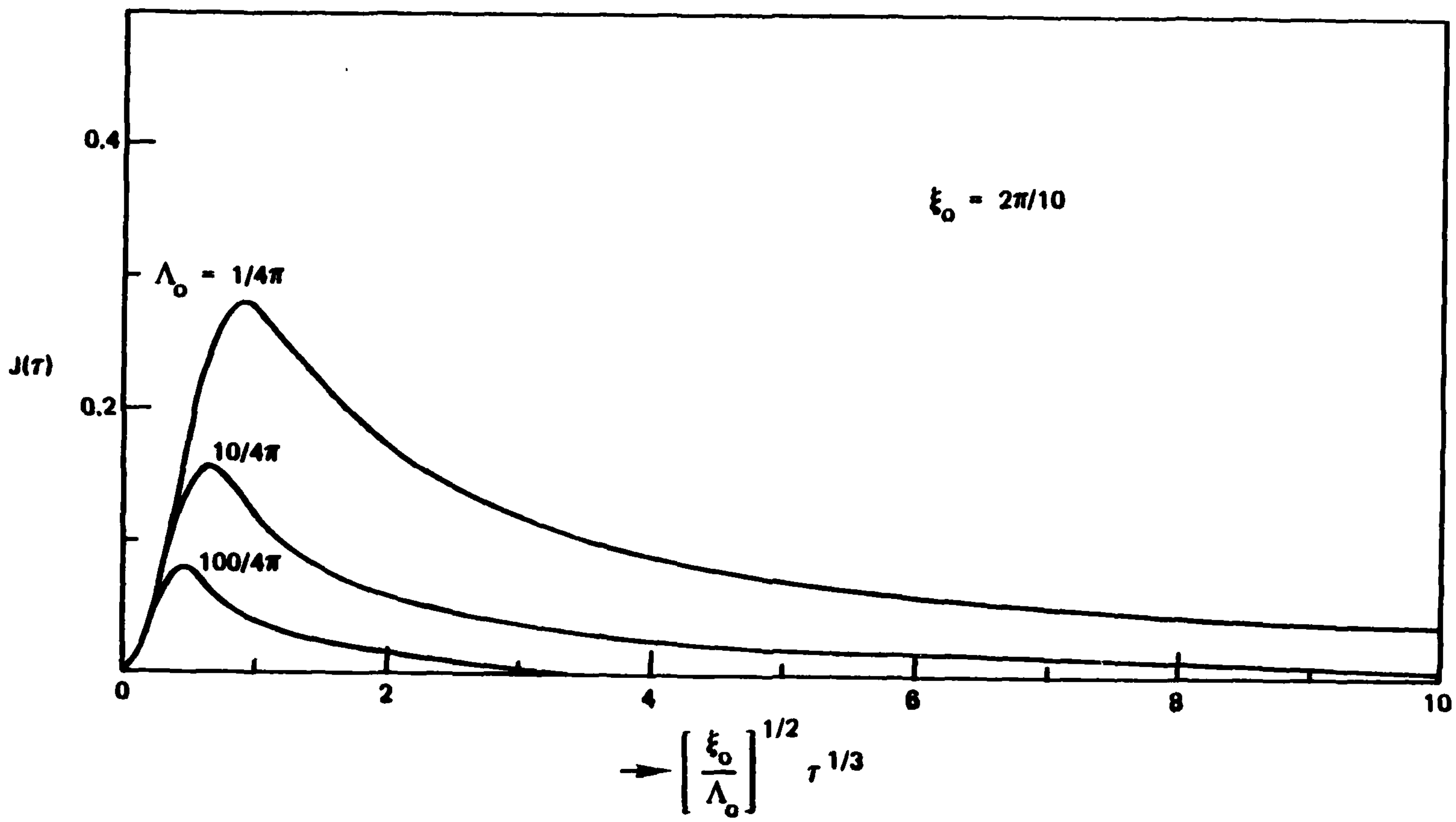

FIG. 3: Current transient $J(\tau)$ of detonation shock flow versus $(\xi_o/\Lambda_o)^{1/2}\tau^{1/3}$ for $\xi_o = 2\pi/10$, and $\Lambda_o = 1/4\pi$; $10/4\pi$; $100/4\pi$.

CONCLUSION

An elementary, one-dimensional theory for an explosion driven induction generator has been developed, in which the upstream ends of the electrodes are connected by an external R_o-L_o load circuit. The formulas derived for the current I(t) and voltage U(t) permit an optimum generator design by appropriate selection of the system geometry, flow, and load parameters.

The jet flow with (about) constant shock speed $v(t) \approx v_o$ produces an order-of-magnitude more energy than a comparable detonation flow with shock speed $v(t) \propto t^{-1/3}$ decreasing with time. Since shock speeds increasing with time are not realizable in the coplanar duct, the jet flow is ideal for power generation.

The magnetic self-field $\vec{B}(t)$ of the generated current I(t) is of the same order-of-magnitude as the external magnetic field $\vec{B}_o$. $\vec{B}(t)$ considerably reduces the generator performance if the electrodes are connected at their upstream ends by the external load circuit ($\vec{B}(t)$ and $\vec{B}_o$ antiparallel).

An optimum power output is achieved by attaching the external load circuit to the downstream ends of the electrodes so that $\vec{B}(t)$ and $\vec{B}_o$ are in the same direction. For a maximum energy output the parameters ξ_o, I_o, $\hat{t}$ should be chosen as large and Λ_o as small as possible.

For the transit period $\hat{t}$ of the shockwave, explosion driven generators are generally contained by inertial confinement. For this reason, no attempt has been made to discuss generator performance for times beyond $\hat{t}$.

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DETONATION-DRIVEN INDUCTION GENERATOR WITH POSITIVE SUPERPOSITION OF EXTERNAL AND INDUCED MAGNETIC FIELDS

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ABSTRACT

Based on Maxwell's equations, the electric current and voltage pulses induced in explosion-driven induction generators ($R_m = \mu_o \sigma v L >> 1$) with plane electrodes and homogeneous external magnetic field $\vec{B}_o$ are calculated in closed form for an external load circuit with resistance R_o and inductance L_o con- connected to the downstream ends of the electrodes so that the magnetic self-field $\vec{B}(t)$ of the generated current $I(t)$ is in the same direction as the transverse external field $\vec{B}_o$. Two plasma shock flow models are considered: (i) the jet flow with shock speed $v(t) = v_o H(t)$ and (ii) the plane detonation flow with shock speed $v(t) = (2/3)(E_o/\rho_o)^{1/3} t^{-1/3}$. It is shown that the jet flow produces large cumulative current and voltage pulses which increase with time, whereas the detonation flow generates considerably smaller and shorter current and voltage pulses. In both cases, the magnetic self-field $\vec{B}(t)$ of the generated current $I(t)$ is of the order of magnitude of $\vec{B}_o$. In case of positive superposition of the induced magnetic field on the external magnetic field $\vec{B}_o$ maximum flux changes are obtained. For this reason, in induction generators the external load circuit should not be attached to the upstream ends or the centers, but to the downstream ends of the electrodes.

INTRODUCTION

Induction generators are based on the electromagnetic interaction of an electrically conducting flow field with a magnetic field, where at least one of the fields is transient so that voltages are induced in an external circuit which are due to temporal magnetic flux changes $d\Phi/dt \neq 0$. An explosion-driven induction generator with a transient plasma shock flow in a constant external magnetic field $\vec{B}_o$ has been analyzed[1] when the external $R_o - L_o$ load circuit is attached to the flow entrance ends of the electrodes. In this case, the self-magnetic field $\vec{\tilde{B}}(t)$ of the generated current I(t) is antiparallel to the transverse external magnetic field $\vec{B}_o$ so that the magnetic flux changes $d\Phi/dt$ during the transit time $\hat{t}$ of the flow are reduced.[1] Herein, a novel explosion-driven induction generator with optimum magnetic flux changes is proposed, in which the external $R_o - L_o$ load circuit is connected to the flow exit ends of the coplanar electrodes so that the induced magnetic field $\vec{\tilde{B}}(t)$ of the current I(t) in the shock front current layer and the electrodes can superimpose themselves positively on the external magnetic field $\vec{B}_o$. This effect is quantitatively significant since $\vec{\tilde{B}}(t)$ and $\vec{B}_o$ are parallel and of the same order of magnitude. Thus, considerably larger current, voltage, and power pulses are induced than in the antiparallel $\vec{B}_o - \vec{\tilde{B}}$ case.

In induction generator experiments, the external $R_o - L_o$ load circuit is usually attached to the center of the electrodes.[2,3,4] In earlier experiments, a detonation-produced plasma with a decreasing shock velocity $v(t) \rightarrow 0$ for $t \rightarrow \infty$ is directly injected into the inter-electrode space,[2] whereas, in more recent experiments,[3,4] discontinuous plasma jets are driven across an external, constant magnetic field $\vec{B}_o$. Plasma jets $v(t) \approx v_o H(t)$ of nearly constant shock speed v_o are produced, e.g., by implosion of a dense noble gas between flyer plates and subsequent expansion through a diaphragm into the generator channel,[3] or by means of a tube-like arrangement of the solid explosive resulting in a cylindrical implosion with large axial shock velocity of the filling gas.[4]

Detonation-driven induction generators have been operated with shock velocities $v = 10^4 - 10^5$ m/sec, external magnetic fields $B_o = 1 - 10$ Tesla, conductivities $\sigma > 10^4\ \Omega^{-1}/\mathrm{m}$, and flow energies $\frac{1}{2}\rho v^2 \sim 10^{10} - 10^{12}$ Joule m^{-3} at pressures $p > 10^4$ bar ($\rho > 10^2$ kg m^{-3}). The corresponding magnetic Reynolds number R_m is large, whereas the magnetic interaction number R_i is small:

$$R_m = \mu_o \sigma v L >> 1 \quad , \quad R_i = B_o^2/\mu_o \rho v^2 << 1 \quad , \tag{1}$$

where $L \sim |\nabla\times\vec{B}/B|$. For $R_m >> 1$, the electric current is restricted to a thin layer $\delta \sim (\hat{t}/\mu_o\sigma)^{1/2}$ behind the shock front ($\hat{t}$ = transit time of flow in $\vec{B}_o$). In view of the large dynamic pressure $\frac{1}{2}\rho v^2$ compared to the magnetic pressure $B_o^2/2\mu_o = 10^6 - 10^8$ Joule m^{-3}, the efficiency of energy transformation is small, while the absolute performance is considerable since currents $I \sim (B_o/\mu_o)b > 10^4$ amps (b = duct width) and energies $\overline{UI\hat{t}} > 10^4$ Joule have been obtained experimentally per detonation.[3,4]

In the following, the initial-value problem for a detonation-driven generator is solved in closed form by means of Maxwell's equations, in which the downstream ends ($x = x_o$) of the coplanar electrodes are connected by an external $L_o - R_o$ load circuit. In this case, solutions for the magnetic field $\vec{B}(t)$ of the generated currents $I(t)$ exist which are parallel to the external magnetic field $\vec{B}_o$ (Fig. 1). Two types of plasma shock flows are considered corresponding to i) the idealized jet flow with constant shock speed v_o and ii) the plane detonation flow due to an explosive energy release E_o per unit area in a gas of initial density ρ_o:

$$v(t) = v_o H(t) \quad , \quad \hat{x}(t) = v_o t H(t) \quad , \tag{2}$$
$$0 < t \leqslant \hat{t} \quad , \quad \hat{t} = x_o/v_o \quad ,$$

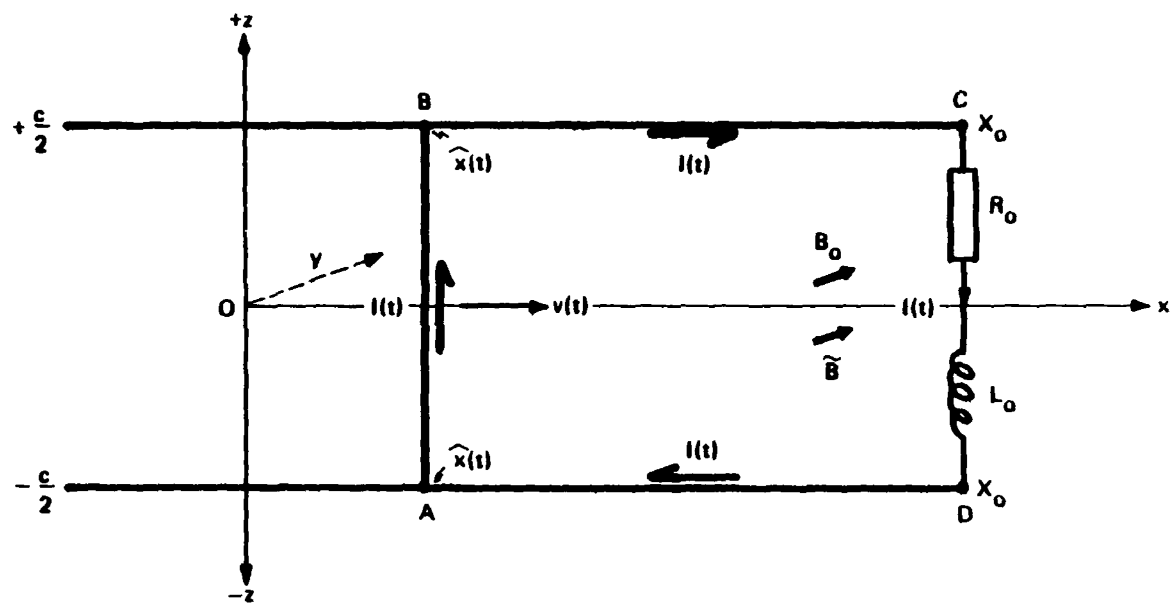

FIG. 1: Generator model with plane electrodes $z = \pm c/2$, applied magnetic field $\vec{B}_o$, and $R_o - L_o$ load circuit CD. Shock front AB at $x = \hat{x}(t) > 0$.

and

$$v(t) = (2/3)(E_o/\rho_o)^{1/3}t^{-1/3} \quad , \quad \hat{x}(t) = (E_o/\rho_o)^{1/3}t^{2/3} \quad , \qquad (3)$$

$$0 \leqslant t \leqslant \hat{t} \quad , \quad \hat{t} = (E_o/\rho_o)^{-1/2}x_o^{3/2} \quad ,$$

where $\hat{x}(t)$ is the shock front position, and $\hat{t}$ is the transit time of the shock front $\hat{x}(t)$ in the duct of effective length x_o. The self-similar solution (3) is asymptotically correct for larger times t,[5] but can be used for t > 0 since the singularity of v(t) at t = 0 is without physical consequences for generator applications.

The mechanical stability of the generator during the shock transit time $\hat{t}$ is provided for by inertial confinement of the duct through high density plaster. No attempt is made at discussing the electrical output for times $t > \hat{t}$ of the explosive disintegration of the generator. The external circuit location x_o is an effective one, which can be calculated once the geometries of the inductance L_o and resistance R_o are specified.

INITIAL-VALUE PROBLEM

Figure 1 exhibits a one-dimensional model for a detonation-driven generator in which the shock wave flow $v(t) = d\hat{x}(t)/dt$ is initiated at time $t = 0$ in the plane $x = 0$ by explosive charges in the space $x < 0$ so that the shock front is at $x = \hat{x}(t) \geqslant 0$ for $t \geqslant 0$. The copper electrodes in the planes $z = \pm c/2$ have the extensions $x > x_o < \infty$ and $-b/2 < y < +b/2$ and are connected by an external circuit CD with load R_o and inductance L_o at the flow exit end $x = x_o$. The entire system is embedded in a homogeneous magnetic field $\vec{B}_o = \{0,B_o,0\}$. As a result of the $\vec{v} \times \vec{B}_o$ interaction of the conducting shock front ($\sigma > 10^4\ \Omega^{-1}/m$) with the external magnetic field $\vec{B}_o$, a current $I(t)$ is induced behind the shock front AB in a cross section $\Delta x \Delta y = b\delta$, where $\sigma \sim (\hat{t}/\mu_o\sigma)^{1/2}$ is the (small) width of the current layer. The current $I(t)$ flows in the closed loop $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$, where the leg AB at $x = \hat{x}(t)$ moves with the shock speed $v(t) = d\hat{x}(t)/dt$.

Application of the surface curl equation $\vec{n} \times [\vec{B}] = \mu_o\vec{I}(t)/b$ to the interfaces AB, BC, CD, DA demonstrates — in the one-dimensional approximation neglecting end effects at $y = \pm b/2$ and $x = x_o$ — that the current $I(t)$ produces a magnetic self-field in the extending loop volume,

$$\tilde{\vec{B}}(t) = \{0,\mu_o I(t)/b,0\} \quad , \quad \hat{x}(t) < x < x_o \quad , \quad |y| < \frac{b}{2} \quad , \quad |z| < \frac{c}{2} \quad , \tag{4}$$

which is parallel to the external magnetic field $\vec{B}_o$ if $I(t) > 0$. The total magnetic field within the loop volume is the positive superposition

$$\vec{B}(t) = \{0,B_o+\mu_o I(t)/b,0\} \quad , \quad \hat{x}(t) < x < x_o \quad , \quad |y| < \frac{b}{2} \quad , \quad |z| < \frac{c}{2} \, , \tag{5}$$

whereas $\vec{B}(t) = \vec{B}_o$ outside of this region. Integration of the Maxwell equation $\nabla \times \vec{E} = -\partial\vec{B}/\partial t$ over the cross section ABCD of the extending current loop yields by Stokes' theorem

$$\oint \vec{E}^* \cdot d\vec{r} = - \oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint (\vec{v} \times \vec{B}) \cdot d\vec{r} \quad , \tag{6}$$

where

$$\vec{E}^* = \vec{E} + \vec{v} \times \vec{B} \quad , \quad v^2 << 1/\mu_o \varepsilon_o \quad , \tag{7}$$

and $\vec{v}(t)$ is the velocity field of the sections of the closed loop, at the point corresponding to the path element $d\vec{r}$ [$\vec{v}(t) = \vec{e}_x d\hat{x}(t)/dt$ for AB, but $\vec{v}(t) = \vec{0}$ for CD, DA, CB]. The surface element $d\vec{S} = +\vec{e}_y cdx$ forms with $d\vec{r}$ [pointing everywhere in the direction $\vec{I}(t)$], a right-hand system. By Eqs. (6) and (7)

$$(R_o + R_p)I(t) + L_o \frac{dI(t)}{dt} = -c \int_{\hat{x}(t)}^{x_o} \frac{d\tilde{B}(t)}{dt} dx + \int_{-c/2}^{+c/2} \frac{d\hat{x}(t)}{dt} [B_o + \tilde{B}(t)] dz \quad , \tag{8}$$

since $\int_A^B E^* dz = IR_p$ (R_p = plasma resistance), $\int_C^D Edz = R_o I + L_o dI/dt$, and $\int Edz \doteq 0$ for the sections BC and DA of the highly conducting electrodes ($\sigma_{Cu} \sim 10^8 \ \Omega^{-1}/m$). Accordingly,

$$(R_o + R_p)I(t) + \frac{d}{dt} \{[L_o + \tilde{L}(t)]I(t)\} = cB_o \frac{d\hat{x}(t)}{dt} \quad , \tag{9}$$

where

$$\tilde{L}(t) = \mu_o c[x_o - \hat{x}(t)]/b \geq 0 \quad , \quad 0 \leq \hat{x}(t) \leq x_o \tag{10}$$

is the time-dependent inductance of the electrode sections AD and BC of decreasing length $\Delta x = x_o - \hat{x}(t)$. Multiplication of Eq. (9) by I(t) leads to the energy conservation law

$$(R_o + R_p)I^2 + \frac{d}{dt} \{\frac{1}{2} [L_o + \tilde{L}(t)]I^2\} = \frac{\mu_o c}{2b} I^2 \frac{d\hat{x}(t)}{dt} + I \frac{d\hat{x}(t)}{dt} B_o c \quad . \tag{11}$$

The plasma resistance $R_p = R_p(t)$ is a slowly varying function of t and can be calculated only via a detailed physical model of the current sheath δ at $x = \hat{x}(t)$. However, since R_o is sufficiently large in actual experiments, $R_o + R_p$ can be treated as a quasi-constant,

$$R_o + R_p(t) \cong \text{constant} \quad , \quad R_o > R_p \quad ; \quad R_o + R_p(t) = R_o \quad , \quad R_o >> R_p \quad . \tag{12}$$

For a generalized formulation, dimensionless dependent and independent variables are introduced by

$$J(\tau) = I(t)/I_o \quad , \quad \tau = t/t_o \quad , \quad \hat{\xi}(\tau) = \hat{x}(t)/c \tag{13}$$

with

$$I_o = (B_o/\mu_o)b \quad , \quad t_o = (\mu_o c^2/b)/(R_o + R_p) \quad , \quad \Lambda_o = (x_o/c) + L_o/(\mu_o c^2/b) \quad . \tag{14}$$

By Eqs. (9), (10), (13), and (14), the electric current I(t) in the explosion-driven generator with positive external and induced magnetic field superposition is determined by the inhomogeneous initial-value problem with variable coefficients:

$$\frac{d}{d\tau}\{[\Lambda_o - \hat{\xi}(\tau)]J(\tau)\} + J(\tau) = \frac{d\hat{\xi}(\tau)}{d\tau} \quad , \quad 0 < \tau < \hat{\tau} \quad , \tag{15}$$

$$J(\tau = 0) = 0 \quad . \tag{16}$$

Equations (15) and (16) are solved for the jet and detonation flows given by Eqs. (2) and (3), respectively, which define the dimensionless coordinate $\hat{\xi}(\tau)$ of the shock front.

In view of the large flow energy, $\frac{1}{2}\rho v^2 >> B_o^2/2\mu_o$, velocity decreases due to Lorentz and viscous forces are neglected in the above generator model. Furthermore, viscous boundary layers are disregarded since the plasma moves in the form of a rectangular velocity slug through the duct in large Reynolds number flow, $R = \rho vL/\mu >> 1$.

JET-FLOW SOLUTION

For the jet flow, the velocity and coordinate of the shock front are given in Eq. (2). The dimensionless shock front coordinate $\hat{\xi}(\tau)$, ξ_o, and the transit time $\hat{\tau}$ are, by Eqs.(13) and (2),

$$\hat{\xi}(\tau) = \xi_o \tau \quad , \quad \xi_o = \frac{v_o}{c} \left(\frac{\mu_o c^2/b}{R_o + R_p} \right) \quad , \quad \hat{\tau} = \frac{x_o}{v_o} \left(\frac{R_o + R_p}{\mu_o c^2/b} \right) \quad . \tag{17}$$

The initial-value problem (15) and (16) has for $\hat{\xi}(\tau) = \xi_o\tau$ a closed-form solution, which is well behaved:

$$J(\tau) = \frac{\xi_o}{1 - \xi_o} [1 - (1 - \frac{\xi_o}{\Lambda_o} \tau)^{(1-\xi_o)/\xi_o}] \quad , \quad 0 \leq \tau \leq \hat{\tau} \quad , \tag{18}$$

where

$$\lim_{\xi_o \to 1} J(\tau) = - \ln(1 - \frac{1}{\Lambda_o} \tau) < \infty \quad , \qquad \frac{1}{\Lambda_o} \tau < 1 \quad . \tag{19}$$

The critical time $\tilde{\tau}$ defined by $\xi_o\tilde{\tau}/\Lambda_o = 1$ is not attainable within the transit period $\hat{\tau}$ since

$$\tilde{\tau} \equiv \frac{\Lambda_o}{\xi_o} = (1 + \frac{L_o}{\mu_o c x_o/b})\hat{\tau} \quad , \quad \hat{\tau} < \tilde{\tau} \text{ for } L_o > 0 \quad . \tag{20}$$

For short times $\tau \to 0$ and the maximum time $\hat{\tau}$, Eq. (18) gives, respectively, the limits

$$J(\tau) = (\xi_o/\Lambda_o)\tau \quad , \quad (\xi_o/\Lambda_o)\tau \ll 1 \quad , \tag{21}$$

$$J_{max} = \frac{\xi_o}{1 - \xi_o} [1 - (1 - \frac{\xi_o}{\Lambda_o} \hat{\tau})^{(1-\xi_o)/\xi_o}] \quad , \quad \tau = \hat{\tau} \quad . \tag{22}$$

Equation (22) represents the maximum generator current for arbitrary $\xi_o > 0$ and leads to the special formulas

$$J_{max} \cong \xi_o/(1 - \xi_o) \text{ for } \xi_o < 1 \quad , \quad L_o << \mu_o c x_o/b \quad , \tag{23}$$

$$J_{max} = -\ln(1 - \frac{1}{\Lambda_o}\hat{\tau}) < \infty \quad \text{for } \xi_o = 1 \quad . \tag{24}$$

The generated voltage, $U = R_oI + L_o dI/dt$, is by Eq. (18) in dimensionless representation ($U_o = I_oR_o$)

$$\mathcal{U}(\tau) = \frac{\xi_o}{1 - \xi_o}[1 - (1 - \frac{\xi_o}{\Lambda_o}\tau)^{(1-\xi_o)/\xi_o}] + \frac{L_o/R_o}{t_o}\frac{\xi_o}{\Lambda_o}(1 - \frac{\xi_o}{\Lambda_o}\tau)^{(1-2\xi_o)/\xi_o} \quad ,$$
$$0 \leq \tau \leq \hat{\tau} \quad , \tag{25}$$

where

$$\lim_{\xi_o \to 1} \mathcal{U}(\tau) = -\ln(1 - \frac{1}{\Lambda_o}\tau) + \frac{L_o/R_o}{t_o}\Lambda_o^{-1}(1 - \Lambda_o^{-1}\tau)^{-1} < \infty, \quad 0 \leq \tau \leq \hat{\tau} \tag{26}$$

and

$$\mathcal{U}(\tau) = [1 + \frac{L_o/R_o}{t_o}(\frac{1 - 2\xi_o}{\Lambda_o})]\frac{\xi_o}{\Lambda_o}\tau + \frac{L_o/R_o}{t_o}\frac{\xi_o}{\Lambda_o} \quad , \quad \frac{\xi_o}{\Lambda_o}\tau << 1 \quad , \tag{27}$$

$$\mathcal{U}_{max} = \frac{\xi_o}{1 - \xi_o}[1 - (1 - \frac{\xi_o}{\Lambda_o}\hat{\tau})^{(1-\xi_o)/\xi_o}] + \frac{L_o/R_o}{t_o}\frac{\xi_o}{\Lambda_o}(1 - \frac{\xi_o}{\Lambda_o}\hat{\tau})^{(1-2\xi_o)/\xi_o} \quad ,$$
$$\tau = \hat{\tau} \quad . \tag{28}$$

For special cases, the maximum voltage is given by simpler formulas,

$$\mathcal{U}_{max} \cong \xi_o/(1 - \xi_o) \text{ for } \xi_o < 1 \quad , \quad L_o << \mu_o c x_o/b \quad , \tag{29}$$

$$\mathcal{U}_{max} = -\ln(1 - \Lambda_o^{-1}\hat{\tau}) + \frac{L_o/R_o}{t_o}\Lambda_o^{-1}(1 - \Lambda_o^{-1}\hat{\tau})^{-1} < \infty \text{ for } {}_o = 1 \quad . \tag{30}$$

Example. For $b = 10^{-1}$ m, $c = 1$ m, $x_o = 1$ m, $\sigma = 10^5\ \Omega^{-1}/\text{m}$, $R_p \cong \sigma^{-1}c/b\delta = 10^{-2}\ \Omega$, $R_o + R_p = 2 \times 10^{-1}\ \Omega$, $L_o = 4\pi \times 10^{-6}$ Henry, $v_o = 10^4$ m/sec, $B_o = 1$ Tesla, the characteristics of the generator are: $t_o = 2\pi \times 10^{-5}$ sec, $\hat{t} = 10^{-4}$ sec, $\xi_o = 2\pi/10$, $\Lambda_o = 2$, $I_o = 10^6/4\pi$ amps, $I_{max} \cong 1 \times 10^5$ amps, $U_{max} \cong 2 \times 10^4$ volts, $(UI)_{max} \cong 2 \times 10^9$ watts, $\Delta E = \overline{UI}\hat{t} \sim 10^5$ joules.

Figure 2 shows the current $J(\tau)$ versus $(\xi_o/\Lambda_o)\tau$ for $\xi_o = 2\pi/10$ and $\xi_o = 9/10$ in the case of $L_o << \mu_o c x_o/b$ so that $\hat{\tau} \lesssim \tilde{\tau}$ based on Eq. (18). While for $\xi_o = 2\pi/10$ the current rises steadily to $J_{max} = 2\pi/(10 - 2\pi) = 1.690$, $J(\tau) \sim 1$ nearly throughout the interval $0 < \tau < \hat{\tau}$ for $\xi_o = 9/10$, and the sharp rise to $J_{max} = 9$ occurs not until the very end $\tau \lesssim \hat{\tau}$ of the flow transit period. It should be noted that the generator produces in each case average currents $\bar{I} > I_o$ in excess of the normalization current $I_o = (B_o/\mu_o)b$. Experimental current oscillograms are not available for comparison, since the advantages of a generator with a load circuit attached to the downstream ends of the electrodes have apparently not yet been recognized.

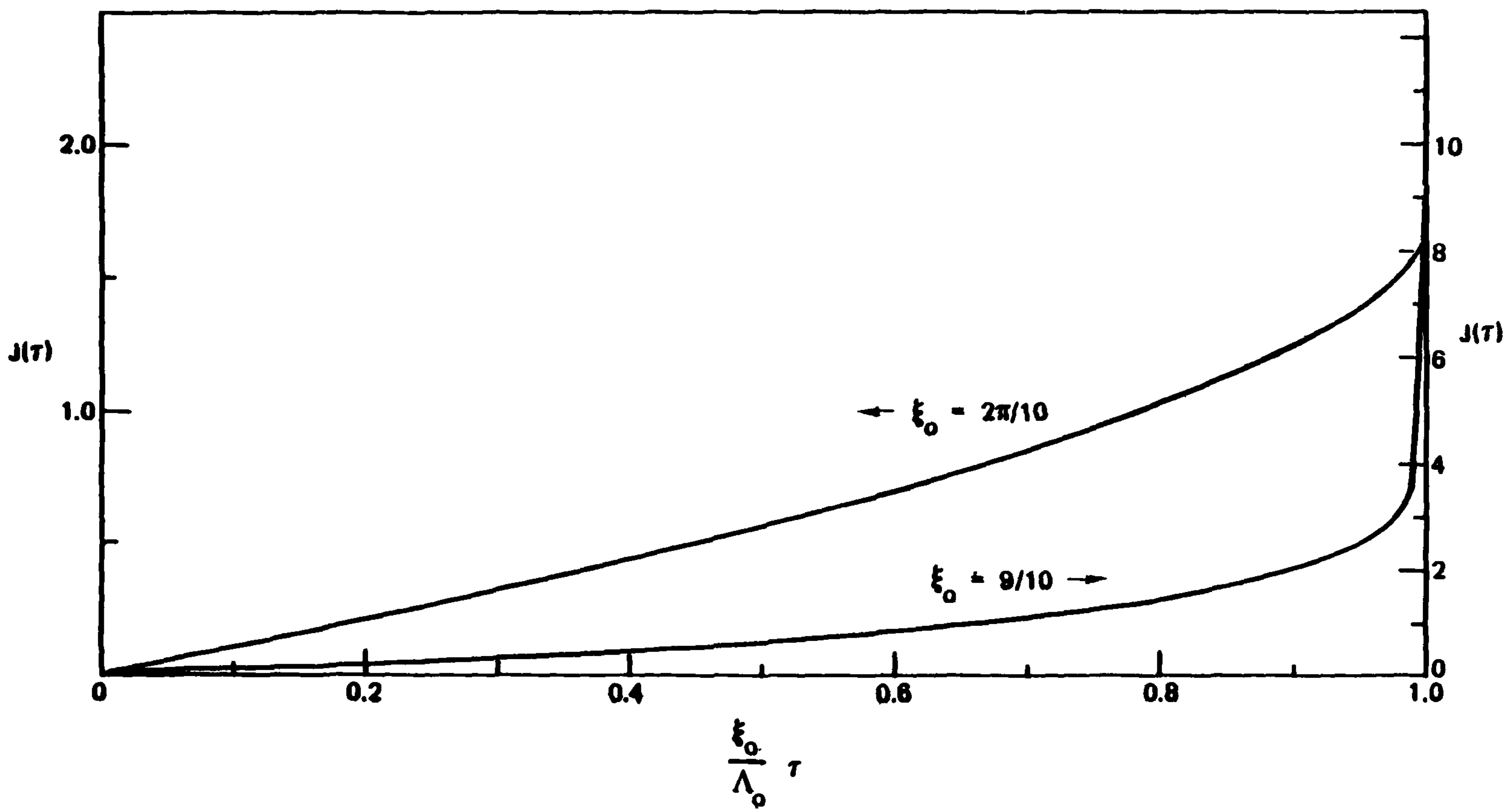

FIG. 2: Current transient $J(\tau)$ of jet shock flow versus $(\xi_o/\Lambda_o)\tau$ for $\xi_o = 2\pi/10$; $9/10$.

DETONATION FLOW SOLUTION

For the detonation flow, the velocity and coordinate of the shock front are given in Eq. (3). The dimensionless shock front coordinate $\hat{\xi}(\tau)$, ξ_o, and the transit time $\hat{\tau}$ are, by Eqs. (13) and (3),

$$\xi(\tau) = \xi_o \tau^{2/3} \quad , \quad \xi_o = c^{-1}\left(\frac{E_o}{\rho_o}\right)^{1/3}\left(\frac{\mu_o c^2/b}{R_o + R_p}\right)^{2/3} \quad , \quad \hat{\tau} = \xi_o^{-3/2}\left(\frac{x_o}{c}\right)^{3/2} \quad . \tag{31}$$

The initial-value problem (15) and (16) has for $\hat{\xi}(\tau) = \xi_o \tau^{2/3}$ a well-behaved solution involving an indefinite integral with regular integrand:

$$J(\tau) = -1 + \frac{1 + \Lambda_o^{-1}\int_0^{\tau} [(1 + \alpha_o \tau^{1/3})/(1 - \alpha_o \tau^{1/3})]^{3/2\alpha_o \xi_o} e^{-3\xi_o^{-1}\tau^{1/3}} d\tau}{(1 - \alpha_o^2 \tau^{2/3})[(1 + \alpha_o \tau^{1/3})/(1 - \alpha_o \tau^{1/3})]^{3/2\alpha_o \xi_o} e^{-3\xi_o^{-1}\tau^{1/3}}} \quad ,$$

$$0 < \tau < \hat{\tau} \quad , \tag{32}$$

where

$$\alpha_o \equiv (\xi_o/\Lambda_o)^{1/2} \quad . \tag{33}$$

The critical time $\tilde{\tau}$ defined by $\alpha_o \tilde{\tau}^{1/3} = 1$ lies always outside the transit period $\hat{\tau}$ since

$$\tilde{\tau} \equiv \alpha_o^{-3} = \left(1 + \frac{L_o}{\mu_o c x_o/b}\right)^{3/2} \hat{\tau} \quad , \quad \hat{\tau} < \tilde{\tau} \text{ for } L_o > 0 \quad . \tag{34}$$

For short times $\tau \to 0$ and the maximum time $\hat{\tau}$, Eq. (32) gives, respectively, the limits

$$J(\tau) = \alpha_o^2 \tau^{2/3} \quad , \quad \alpha_o \tau^{1/3} << 1 \quad , \tag{35}$$

$$J_{max} = J(\hat{\tau}), \ \tau = \hat{\tau} \quad , \tag{36}$$

where $J(\hat{\tau})$ is given by Eq. (32). In particular, for small inductances L_o,

$$J_{max} \doteq \xi_o/[\tfrac{3}{2}(\Lambda_o/\xi_o)^{1/2} - \xi_o] \text{ for } \xi_o \neq (\tfrac{3}{2})^{2/3}\Lambda_o^{1/3} \quad , \quad L_o << \mu_o c x_o/b \quad . \tag{37}$$

Eq. (32) has positive, monotonically increasing solutions $J(\tau) > 0$ for $\tau \,\varepsilon(0,\hat{\tau})$. The magnetic field $\vec{B}(t)$ of this current is in the same direction as the external magnetic field $\vec{B}_o$ (positive superposition).

The generated voltage, $U = R_o I + L_o dI/dt$, is by Eq. (32) in dimensionless representation ($U_o = I_o R_o$),

$$\mathcal{U}(\tau) = J(\tau) + \frac{L_o/R_o}{t_o}\left\{\frac{(2/3)\xi_o - [\tau^{1/3} - (2/3)\xi_o]J(\tau)}{\Lambda_o(1 - \alpha_o^2\tau^{2/3})\tau^{1/3}}\right\} \quad , \quad 0 < \tau < \hat{\tau} \quad . \tag{38}$$

The inductive singularity, $\mathcal{U}(\tau) \to \infty$ for $\tau \to 0$, is due to the divergence of the self-similar speed,[5] $d\xi(\tau)/d\tau = (2\xi_o/3)\tau^{-1/3} \to \infty$ for $\tau \to 0$, and has, therefore, no physical meaning.

<u>Example</u>. For $b = 10^{-1}$ m, $c = 1$ m, $x_o = 1$ m, $\sigma = 10^5\ \Omega^{-1}$/m, $R_p \doteq \sigma^{-1}c/b\delta = 10^{-2}\ \Omega$, $R_o + R_p = 2 \times 10^{-1}\ \Omega$, $L_o = 4\pi \times 10^{-6}$ Henry, $v(t = 10^{-6}$ sec$) = 2.650 \times 10^4$ m/sec, $B_o = 1$ Tesla, the generator characteristics are: $t_o = 2\pi \times 10^{-5}$ sec, $\hat{t} = 1.262 \times 10^{-4}$ sec, $\xi_o = 2\pi/10$, $\Lambda_o = 2$, $I_o = 10^6/4\pi$ amps, $I_{max} \doteq 3\times10^4$ amps, $U_{max} \doteq 6\times10^3$ volts, $(UI)_{max} \doteq 2\times10^8$ watts, $\Delta E = \overline{UI}\hat{t} \sim 10^4$ joules.

Figure 3 shows the current $J(\tau)$ versus $(\xi_o/\Lambda_o)^{1/2}\tau^{1/3}$ for $\xi_o = 2\pi/10$, $\xi_o = 9/10$, and $\Lambda_o = 2$ based on Eq. (32). In each case, $J(\tau)$ increases monotonically from 0 to its final value J_{max} at transit time $\hat{\tau}$, where $(\xi_o/\Lambda_o)^{1/2}\tau^{1/3} = (x_o/\Lambda_o c)^{1/2} = \sqrt{2}/2$ for $x_o = 1$ m, $c = 1$ m, and $\Lambda_o = 2$. On the average, the produced currents $\overline{I} < I_o$ are smaller than the normalization current $I_o = (B_o/\mu_o)b$. Comparison with Fig. 2 indicates that $\overline{J}(\tau)$ of the detonation flow is by an order of magnitude smaller than $\overline{J}(\tau)$ of the jet flow.

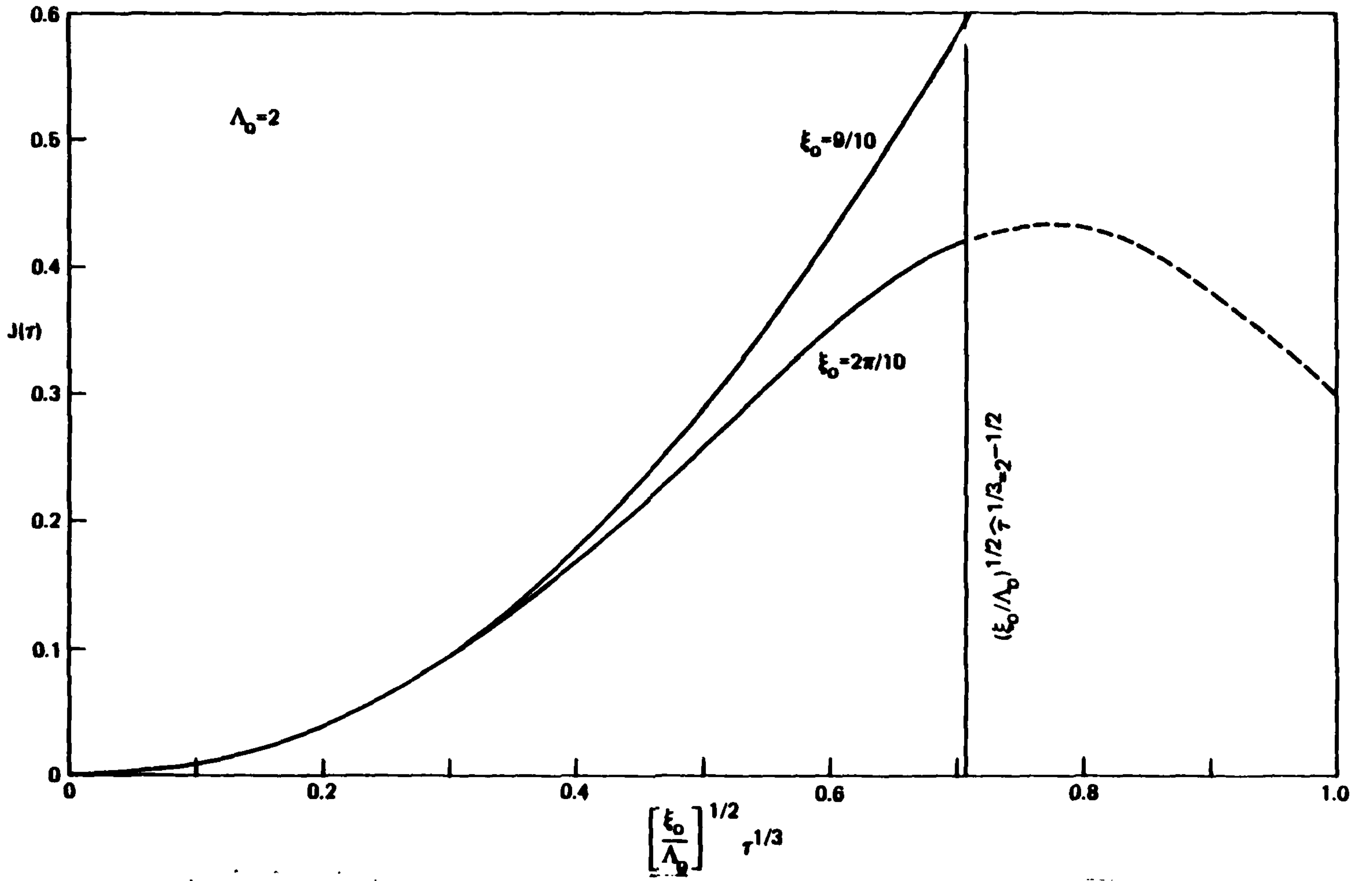

FIG. 3: Current transient $J(\tau)$ of detonation shock flow versus $(\xi_o/\Lambda_o)^{1/2}\tau^{1/3}$ for $\Lambda_o = 2$ and $\xi_o = 2\pi/10;\ 9/10$.

CONCLUSIONS

An electromagnetic theory for an explosion-driven induction generator has been developed in which the downstream ends of the electrodes are connected by an external $R_o - L_o$ load circuit. The formulas derived for the current I(t) and voltage U(t) permit an optimum generator design by appropriate selection of the system geometry, flow, and load parameters.

The jet flow with (about) constant shock speed $v(t) \approx v_o$ produces considerably more energy than a comparable detonation flow with shock speed $v(t) \propto t^{-1/3}$ decreasing with time. Since shock speeds increasing with time are not realizable in the coplanar duct, the jet flow is ideal for power generation.

The magnetic self-field $\tilde{\vec{B}}(t)$ of the generated current I(t) is of the same order of magnitude as the external magnetic field $\vec{B}_o$. The induced magnetic field $\tilde{\vec{B}}(t)$ considerably increases the performance of generators with shock flow, since it superimposes itself positively on the external magnetic field $\vec{B}_o$ if the electrodes are connected at their downstream ends by the external load circuit ($\tilde{\vec{B}}(t)$ and $\vec{B}_o$ in the same direction for optimum magnetic flux changes).

An optimum power output is achieved by (i) attaching the external load circuit to the downstream ends of the electrodes and (ii) driving the generator with a jet shock flow. For a maximum energy output, the parameters ξ_o, I_o, $\hat{t}$ should be chosen as large as possible, and Λ_o should be chosen as small as possible.

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POWER GENERATION BY SHOCKWAVE INTERACTION WITH SOLIDS ELECTRICALLY POLARIZABLE BY STRESS

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Abstract

An elementary theory of the interaction of shock waves with solids which are electrically polarizable by stress is given and discussed with regard to the generation of electric power pulses. The solid is assumed to be a one-dimensional slab between plane electrodes, which are connected through an external load circuit. It is shown that the temporal current build-up in the external circuit, due to electric polarization and electric conduction behind the shock front of the stress wave, space charge and electric polarization relaxations, is determined by an inhomogeneous Volterra integral equation. The latter is solved analytically, and current pulse forms for incident step-shaped stress waves are calculated. For shocked solids with cross section of the order of one square meter, energies of the order $\Delta E \sim 10^5$ Joule per pulse should be achievable.

INTRODUCTION

Although electric energy pulses of 10^4 Joules and larger have been produced in explosion driven induction generators [1-3], the requirement of large external magnetic fields of at least 1 Tesla renders these magnetogasdynamic generators unsuitable for many applications, in particular since the prospects for self-excitation of such magnetic fields during pulse times of the order 10^{-5} sec are not promising. For these reasons, other methods for conversion of chemical energy of explosives into electric energy have to be found. E.g., such a "non-magnetic" power supply could be used to produce the magnetic field required for magnetogasdynamic generators.

Not only piezoelectric crystals such as tourmaline or ferroelectric crystals such as lead circonate titanate, but also a large number of other solids become strongly electrically polarized when subject to pressures $p > 10^4$ bar. Such materials are, e.g., electrets, semiconductors, alkali halides, plexiglas, polysterene [4]. These solids simultaneously develop good to metallic electrical conductivities when subject to pressure $p = 10^4$-10^6 bar, due to nonthermal ionization by overlapping of atomic wave functions at extreme pressures [5]. Similar to the chemical polarization field in a battery, the electric polarization caused by the stress shock wave in the solid drives an electric current pulse through an external circuit attached via electrodes (transverse to the polarization vector) to the solid.

The interaction of a shock wave with solids, which are electrically polarizable by stress, appears to have potential for the generation of electric power pulses. Table I gives the stress polarization coefficient [6] f, the dielectric permittivity [6] $\varepsilon = \varepsilon^* \varepsilon_o$, the density ρ, and (dimensional

TABLE I: Order-of-magnitude estimates of stress polarization P, electric field E, current I, and energy ΔE for a stress polarizable slab ($A = 1\ m^2$, $a = 10^{-2} m$) subject to a shock wave with speed $v = 10^3 m/sec$.

	Quartz	$BaTiO_3$	Rochelle Salt
f[A sec/N]	2.3×10^{-12}	1.0×10^{-10}	4.0×10^{-10}
ρ [kg m^{-3}]	2.65×10^{3}	6.02×10^{3}	1.77×10^{3}
$\hat{\sigma}$ [N/m^2]	1.3×10^{9}	3.1×10^{9}	0.9×10^{9}
ϵ [A sec/Vm]	4×10^{-11}	2×10^{-8}	9×10^{-11}
P[A sec/m^2]	3.0×10^{-3}	3.1×10^{-1}	3.1×10^{-1}
E[V/m]	7.6×10^{7}	1.5×10^{7}	3.9×10^{9}
I[A]	3×10^{2}	3×10^{4}	3×10^{4}
ΔE[J]	1×10^{5}	5×10^{6}	1×10^{8}

analysis) order-of-magnitude estimates of the stress amplitude $\hat{\sigma} \sim \frac{1}{2}\rho v^2$, electric polarization $P \sim \hat{\sigma} f$, electric field $E \sim P/\varepsilon$, short circuit current $I \sim APv/a$, and electric energy $\Delta E \sim (P^2/2\varepsilon)Aa$ for one piezoelectric and two ferroelectric slabs of cross section $A = 1\ m^2$ and thickness $a = 10^{-2}m$ in case of a stress wave with (moderate) speed $v = 10^3 m/sec$. It is seen that E of quartz is of the order of, E of $BaTiO_3$ is below, and E of Rochelle salt is above, the dielectric breakdown field [4]. While the generated voltages $V \sim Ea$ are large, the generated currents I over an area $A = 1/m^2$ are only moderate, but the energy pulses ΔE are significant. Furthermore, these examples show that a stress polarization generator can be operated both below and in the breakdown regime.

The electric polarization of solids by stress shock waves is also of considerable basic physical interest. Attempts at a shock wave interaction theory with stress polarizable solids (short circuited) were made by Allison [7], Zeldovich [8], and Ivanov [9] under various phenomenological assumptions and compared with experimental data from piezoelectric and ferroelectric pressure transducers [4]. The applicability and limitations of these theories have been the subject of extensive discussions [4,7,8,9], which indicate that the problem is not completely understood from first principles.

Herein, a theory of the interaction of stress shock waves with (non-ferroelectric) stress polarizable solids is presented based on Maxwell's equations, when the external circuit contains an ohmic load. It is shown that the electric current pulse I(t) is determined by an inhomogeneous Volterra integral equation of convolution type. Closed-form solutions are obtained for the generated current I(t) by taking into consideration the different time scales at which electric space charge and electric polarization by stress develop in the solid.

The following shock wave interaction theory with solids is in general not applicable to ferroelectric solids (except for rough estimates). Ferroelectrics exhibit large permanent electric polarization and a pronounced hysteresis effect so that the electric displacement $\vec{D} = \vec{D}(\vec{E})$ is no longer a linear but a strongly nonlinear function of the electric field $\vec{E}$ (a hysteresis theory for ferroelectrics is not available).

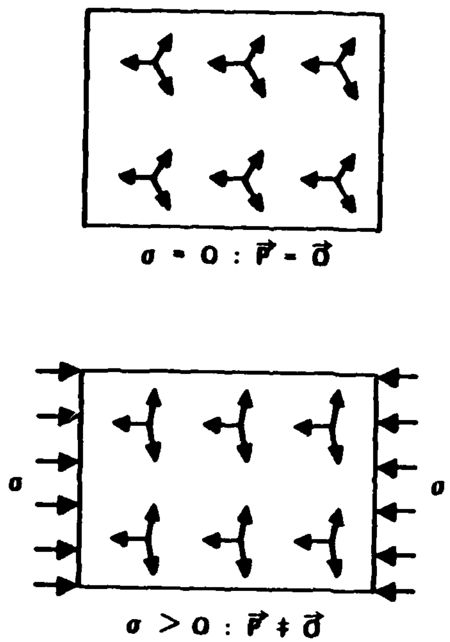

FIG. 1: Electric polarization $\vec{P}$ by stress σ.

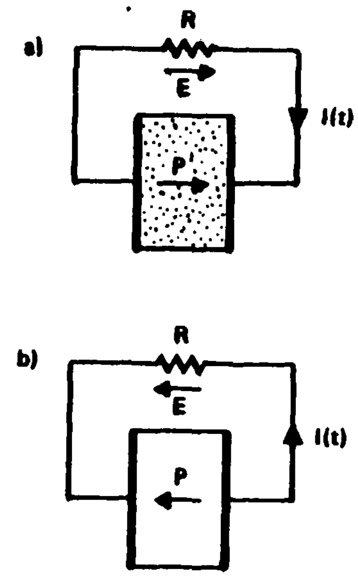

FIG. 2: Direction of circuit current:
a) "+" polarization
b) "–" polarization

POLARIZATION AND CHARGE BUILD-UP

In unstressed, shock polarizable (non-ferroelectric) solids, the electric dipole moments of the molecules add up to a zero vector, whereas under stress the dipoles are displaced preferentially in one direction so that a net polarization results. This is illustrated in Fig. 1a ($\sigma = 0$) and Fig. 1b ($\sigma \neq 0$), which depicts the unstressed and stressed dipole configurations in a quartz crystal subject to a stress component σ which is perpendicular to the optical axis. In Fig. 1b, $\vec{P}$ points to the left, but $\vec{P}$ would point to the right if the crystal were rotated by 180°. For this reason, the polarization vector $\vec{P}$ may point in or opposite to the direction of propagation of the stress wave depending on the chosen orientation of the crystal. For a stress wave $\sigma(x,t)$ propagating in the positive x-direction, the designation "±" polarization is used if $\vec{P}$ points into the ±x-directions. The resulting direction of the current I(t) in the external circuit is shown in Fig. 2a for the "+" polarization, and in Fig. 2b for the "-" polarization, based on Kirchhoff's law $\oint \vec{E}\cdot d\vec{r} = 0$.

The stress polarization generator model (Fig. 3) consists of a shock polarizable solid of cross section A and thickness a between plane electrodes at $x = 0$ and $x = a$, which are connected through an external circuit with load $R > 0$. At time $t = 0$, an explosion produced, step shaped stress wave $\sigma(x,t)$ with amplitude $\hat{\sigma}$ and propagation speed v impinges normal onto the surface $x = 0$ of the solid so that, at time $t > 0$, the shock front is at $x = vt$ and the stress distribution in $0 < x < a$ is

$$\sigma(x,t) = \hat{\sigma}H(vt - x) \quad , \quad t > 0 \quad , \qquad (1)$$

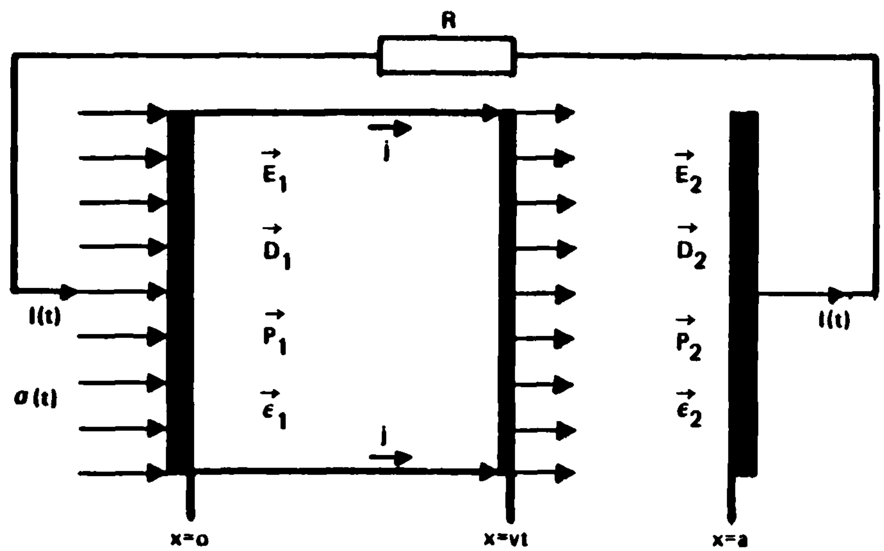

FIG. 3: Shock wave interaction in solid polarized ($\vec{P}_1$) by stress and conducting ($\vec{j}$) behind shock front, $0 < x < vt$.

where

$$H(vt - x) = 1 \quad , \quad 0 < x < vt \qquad , \qquad (2)$$

$$ 0 \quad , \quad vt < x < a \qquad ,$$

is the Heaviside function (compression of the slab $\Delta x = a$ neglected)[7,8,9]. Behind the shock front $x = vt$, the polarization $P(x,t)$ builds up according to a relaxation process with characteristic time τ,

$$\frac{\partial P}{\partial t} + \frac{P}{\tau} = \pm\frac{f}{\tau}\sigma \qquad , \qquad (3)$$

with

$$P(x,t = 0) = 0 \qquad , \qquad (4)$$

as initial condition for the "±" polarizations (Fig. 2a,b), since the solid is initially unpolarized and unstressed. In view of Eq. (1), the solution of Eqs. (3) - (4) is

$$P(x,t) = \pm f\hat{\sigma}[1 - e^{-(vt-x)/v\tau}]H(vt - x) \qquad . \qquad (5)$$

Accordingly, $P(x,t)$ drops exponentially to zero for $x \rightarrow vt$ (shock front), and $P(x,t) = \pm f\hat{\sigma}$ in $0 < x < vt$ for $t >> \tau$. The polarization relaxation time is of the order-of-magnitude $\tau \sim 10^{-6}$ sec.[4]

Due to shock polarization $P(x,t)$ in the stressed region, which is absent in the unstressed region, the electric displacements behind and ahead of the shock front $x = vt$ are [10]

$$D_1 = \varepsilon_1 E_1 + P \quad , \quad 0 < x < vt \qquad , \qquad (6)$$

$$D_2 = \varepsilon_2 E_2 \quad , \quad vt < x < a \qquad , \qquad (7)$$

where [10]

$$\varepsilon_{1,2} = \varepsilon_o(1 + \kappa_{1,2}) \tag{8}$$

so that $\varepsilon_o\kappa_{1,2}E_{1,2}$ are the polarizations due to the electric fields $E_{1,2}(x,t)$. From $\nabla\times\vec{H} = \vec{j} + \partial\vec{D}/\partial t$ follows $\nabla\cdot(\vec{j} + \partial\vec{D}/\partial t) = 0$, and by integration

$$j + \partial D_1/\partial t = I(t)/A \quad , \quad 0 < x < vt \quad , \tag{9}$$

$$\partial D_2/\partial t = I(t)/A \quad , \quad vt < x < a \quad , \tag{10}$$

where

$$j = \sigma E_1 \quad , \quad 0 < x < vt \quad , \tag{11}$$

is the conduction current density behind the shock front $x = vt$ where charge carriers are produced by pressure (conductivity σ).

The x-integration "constant" $I(t)/A$ in Eqs. (9) and (10) is the total (conduction + displacement) current density, i.e., $I(t) \gtrless 0$ is the net current in the external circuit. The space charge convection current density is neglected since it is relativistically small for $v << c$.

From $\nabla\cdot\vec{D}_1 = \rho_1$ and $\nabla\cdot\vec{j} = -\partial\rho_1/\partial t$ result two coupled partial differential equations for the space charge density $\rho_1(x,t)$ and the electric field $E_1(x,t)$ in $0 < x < vt$,

$$\varepsilon_1\frac{\partial E_1}{\partial x} \mp \frac{\hat{\sigma}f}{v\tau}\, e^{-(vt-x)/v\tau}H(vt - x) = \rho_1 \quad , \tag{12}$$

$$\sigma\frac{\partial E_1}{\partial x} = -\frac{\partial\rho_1}{\partial t} \quad , \tag{13}$$

by Eqs. (5), (6), and (11), respectively. The upper and lower signs in Eq. (12) refer to the "±" polarization cases. Accordingly, the space charge build-up in the stressed region is determined by

$$\frac{\partial \rho_1}{\partial t} + \frac{\rho_1}{\tau_o} = \mp \frac{\hat{\sigma} f}{\tau_o v\tau} e^{-(vt-x)/v\tau} H(vt - x) \quad , \quad 0 < x < vt \quad , \tag{14}$$

$$\rho_1(x,t = 0) = 0 \quad . \tag{15}$$

Hence

$$\rho_1(x,t) = \mp \frac{\hat{\sigma} f}{v(\tau - \tau_o)} [e^{-(vt-x)/v\tau} - e^{-(vt-x)/v\tau_o}] H(vt - x) \quad , \tag{16}$$

$$E_1(x,t) - E_1(vt,t) = (v\tau_o/\varepsilon_1)\rho_1(x,t) \quad , \tag{17}$$

for the "±" polarization cases. The space charge relaxation time is defined by

$$\tau_o = \varepsilon_1/\sigma \quad . \tag{18}$$

Substitution of Eq. (17) into Eq. (9) shows that the x-integration "constant" $E_1(vt,t)$ is determined by

$$\frac{dE_1(vt,t)}{dt} + \frac{E_1(vt,t)}{\tau_o} = \frac{I(t)}{A\varepsilon_1} \quad , \tag{19}$$

$$E_1(0,0) = 0 \quad . \tag{20}$$

Hence

$$E_1(vt,t) = (A\varepsilon_1)^{-1} e^{-t/\tau_o} \int_0^t I(t') e^{+t'/\tau_o} dt' \quad . \tag{21}$$

Thus, we obtain the electric fields behind and ahead of the shock front as integral functionals of I(t):

$$E_1(x,t) = \frac{e^{-t/\tau_o}}{A\varepsilon_1} \int_0^t I(t') e^{+t'/\tau_o} dt' \mp \frac{\hat{\sigma} f}{\varepsilon_1(\tau-\tau_o)} [e^{-(vt-x)v\tau} - e^{-(vt-x)/v\tau_o}], \tag{22}$$

$$0 < x < vt \quad ,$$

$$E_2(x,t) = (A\varepsilon_2)^{-1}\int_0^t I(t')dt' \quad , \quad vt < x < a \quad , \tag{23}$$

for the "±" polarization cases. Since $\nabla\cdot\vec{D}_2 = 0$ in the unstressed region, Eq. (23) obtains by direct integration of Eq. (10).

The surface charge density $\rho^*(vt,t) = \vec{n}\cdot[\vec{D}]_{x=vt}$ of the shock front is by Eqs. (5), (6), (7), (22), and (23)

$$\rho^*(vt,t) = (A\varepsilon_2)^{-1}\int_0^t I(t')dt' - (A\varepsilon_1)^{-1}e^{-t/\tau_o}\int_0^t I(t')e^{+t'/\tau_o}dt' \ . \tag{24}$$

It is obvious that in general $\rho^*(vt,t) \neq 0$, i.e., $\rho^*(vt,t) \equiv 0$ only if $\varepsilon_1 = \varepsilon_2$ and $\tau_o \to 0$. Therefore, the assumption $D_1(x,t) = D_2(x,t)$ for $x = vt$ made in the theory of piezoelectric and ferroelectric pressure transducers is in general incorrect.

INTEGRAL EQUATION FOR I(t)

The electric current pulse I(t) in the external circuit is determined by Kirchhoff's law $\oint \vec{E}\cdot d\vec{r} = 0$ for the closed current loop with load R in Fig. 3, where E(x,t) in the solid is given by Eqs. (22) and (23) and $\int \vec{E}_R \cdot d\vec{r} = + IR$. Thus, we obtain for the transient current I(t) the inhomogeneous Volterra integral equation:

$$\frac{vt}{A\varepsilon_1}\int_0^t e^{-(t-t')/\tau_o} I(t')dt' + \frac{a - vt}{A\varepsilon_2}\int_0^t I(t')dt' + RI(t)$$

$$= \pm\frac{\hat{\sigma} f v \tau_o}{\varepsilon_1}\left(1 - \frac{\tau e^{-t/\tau} - \tau_o e^{-t/\tau_o}}{\tau - \tau_o}\right) \quad . \quad (25)$$

The upper/lower signs refer to the "±" polarization cases. A more general representation of this integral equation is achieved by introducing the dimensionless variables

$$J(\mathcal{t}) = I(t)/I_o \quad , \quad I_o \equiv \sigma(\hat{\sigma} f/\varepsilon_1)A \quad ; \quad \mathcal{t} = t/t_o \quad , \quad t_o \equiv \tau \quad ; \quad (26)$$

and

$$\mathcal{a} = a/v\tau \quad , \quad \mathcal{R} = R/(\sigma^{-1}v\tau/A) \quad , \quad \varepsilon = \varepsilon_1/\varepsilon_2 \quad , \quad \delta = \tau_o/\tau \; , \quad (27)$$

as characteristic dimensionless parameters ($\sigma^{-1}v\tau/A$ = internal resistance of solid slab of thickness $v\tau$ traversed by shock during polarization relaxation time τ). By means of Eqs. (26) and (27), the determination of $J(\mathcal{t})$ is reduced to the initial-value problem:

$$\frac{\mathcal{t}}{\delta}\int_0^{\mathcal{t}} e^{-(\mathcal{t}-\mathcal{t}')/\delta} J(\mathcal{t}')d\mathcal{t}' + \varepsilon\frac{\mathcal{a} - \mathcal{t}}{\delta}\int_0^{\mathcal{t}} J(\mathcal{t}')d\mathcal{t}' + \mathcal{R}J(t)$$

$$= \pm\delta\left(1 - \frac{e^{-\mathcal{t}} - \delta e^{-\mathcal{t}/\delta}}{1 - \delta}\right) \quad , \quad (28)$$

$$J(t=0) = 0 \quad . \tag{29}$$

By means of the Laplace transformation and the convolution theorem, the integral equation (28) is reducible to an inhomogeneous differential equation of first order for $\bar{J}(s) = L[J(t)]$ with variable coefficients of not more than third order in s. The general solution can then be obtained by inversion, $I(t) = L^{-1}[\bar{J}(s)]$.

It should be noted that in the derivation of the integral equations (25) and (28), the current I(t) has been assumed to flow in the +x - direction (Fig. 3). Accordingly, one exspects solutions $I(t) < 0$ for "+"polarization and solutions $I(t) > 0$ for "-" polarization.

ANALYTICAL SOLUTION

For electrical power generation, the main interest is in shocked solids with sufficient pressure ionization so that $\delta = \tau_o/\tau << 1$. For this reason, analytical solutions are derived for the case of small space charge relaxation times τ_o, compared with the polarization relaxation time τ. The solution for large space charge relaxation times, $\delta = \tau_o/\tau >> 1$, can also be given in closed form [11].

For fast space charge relaxation, $\tau_o << \tau$, the initial-value problem (28) - (29) reduces for the polarizations $\vec{P}$ in (+) and opposite (-) to the direction of shock propagation to ($\delta << 1$):

$$\varepsilon\frac{a - t}{\delta}\int_0^t J(t')dt' + (R + t)J(t) = \pm\delta(1 - e^{-t}) \quad , \quad (30)$$

$$J(t=0) = 0 \quad . \quad (31)$$

Eq. (30) consists of a cumulative voltage term, the ohmic voltage loss, and a delayed stress polarization voltage source.

By means of the transformation,

$$J(t) = \frac{d}{dt}[\psi(t)e^{(\varepsilon/\delta)t}] \quad , \quad \psi(t=0) = 0 \quad , \quad \psi'(t=0) = 0 \quad , \quad (32)$$

the integral equation problem (30) - (31) is reduced to an initial-value problem for a first order differential equation:

$$(R + t)\frac{d\psi}{dt} + \frac{\varepsilon}{\delta}\,(R + a)\psi = \pm\delta\,(1 - e^{-t})e^{-(\varepsilon/\delta)t} \quad , \qquad (33)$$

$$\psi(t{=}0) = 0 \quad . \qquad (34)$$

Eq. (34) implies $d\psi(t{=}0)dt = 0$ by Eq. (33). The solutions of Eqs. (33) - (34) are for the "±" polarization cases

$$\psi(t) = \pm\delta\,(R + t)^{-\frac{\varepsilon}{\delta}(a+R)} \int_0^t (R + t')^{\frac{\varepsilon}{\delta}(a+R)-1} (1 - e^{-t'})e^{-(\varepsilon/\delta)t'}dt' \,,\quad 0 < t < a. \qquad (35)$$

The corresponding current solutions $J(t)$ for the "±" polarization cases are by Eq. (32):

$$J(t)/\delta = \pm\frac{(1 - e^{-t})}{R + t} \mp \frac{\varepsilon(a - t)}{\delta R^2} \cdot \frac{\int_0^t (1 + \frac{t'}{R})^{\frac{\varepsilon}{\delta}(a+R)-1} (1 - e^{-t'})e^{-(\varepsilon/\delta)t'}dt'}{(1 + \frac{t}{R})^{\frac{\varepsilon}{\delta}(a+R)+1} e^{-(\varepsilon/\delta)t}} \,,$$

$$0 < t < a \quad . \qquad (36)$$

These solutions can be extended to times $t > a$ beyond the transit time $t = a$ if the assumption is made that the shock wave is not reflected at the electrode $x = a$.

Eq. (36) indicates that the integral term decreases much faster with increasing resistance R than the ordinary stress polarization term. The "+" polarization current is - by magnitude - as large as the "-"polarization current,

$$|J(\vec{P})| = |J(\overleftarrow{P})| \quad , \qquad 0 < t < a \quad . \qquad (37)$$

Since the integral term is dominant by magnitude, the solutions of Eq. (36) are $J(t) \gtrless 0$ for the ± polarizations and times $t < a$.

At the transit time $\mathcal{t} = \mathcal{a}$, the integral term in Eq. (36) vanishes, and at this particular instant the current is given by

$$J(\mathcal{t}=\mathcal{a}) = \pm\delta(1 - e^{-\mathcal{a}})/(\mathcal{R} + \mathcal{a}) \quad . \qquad (38)$$

The solution in Eq. (36) shows that the (dimensional analysis) estimate $I \sim APv/a$ (Table I) is correct for $t < a/v$ and $R_a = \sigma^{-1}a/A \sim R$, since $I \sim (\varepsilon\mathcal{a}/\mathcal{R}^2)I_o = (\varepsilon_1/\varepsilon_2)(R_a/R)^2 APv/a$ by Eq. (36). For power applications external resistances $R \sim R_a$ of the order of the slab resistance are of main interest.

Fig. 4 shows computer calculations of current pulses $\mp J(\mathcal{t})$ ("±" polarizations) versus time $0 < \mathcal{t} < \mathcal{a}$ for $\mathcal{R} = 1$ and $\mathcal{a} = 1,2, \ldots 10$ ($\delta/\varepsilon = 10^{-2}$) based on Eq. (36). The pulse height $|J(\mathcal{t})|_{max}$ increases considerably with increasing $\mathcal{a} = a/v\tau$, i.e., if the slab width a is made larger and larger compared with the distance $v\tau$ travelled by the shock wave during the relaxation time τ (saturation for $a \gg v\tau$).

The generated currents $J(\mathcal{t})$ are large for $\mathcal{R} \lesssim 1$, but decrease considerably (about $\propto \mathcal{R}^{-2}$) with increasing $\mathcal{R} > 1$. The steep rise of the current $J(\mathcal{t})$ within a time $\mathcal{t} \le 10^{-1}$, i.e., $t \le \tau/10$ is remarkable. Apparently, the complete build-up of the electric polarization field is not required for the development of the maximum current flow $J(\mathcal{t})$ if $\tau \ll \tau_o$, since the electric polarization fields are already very large for $t \sim \tau/10$.

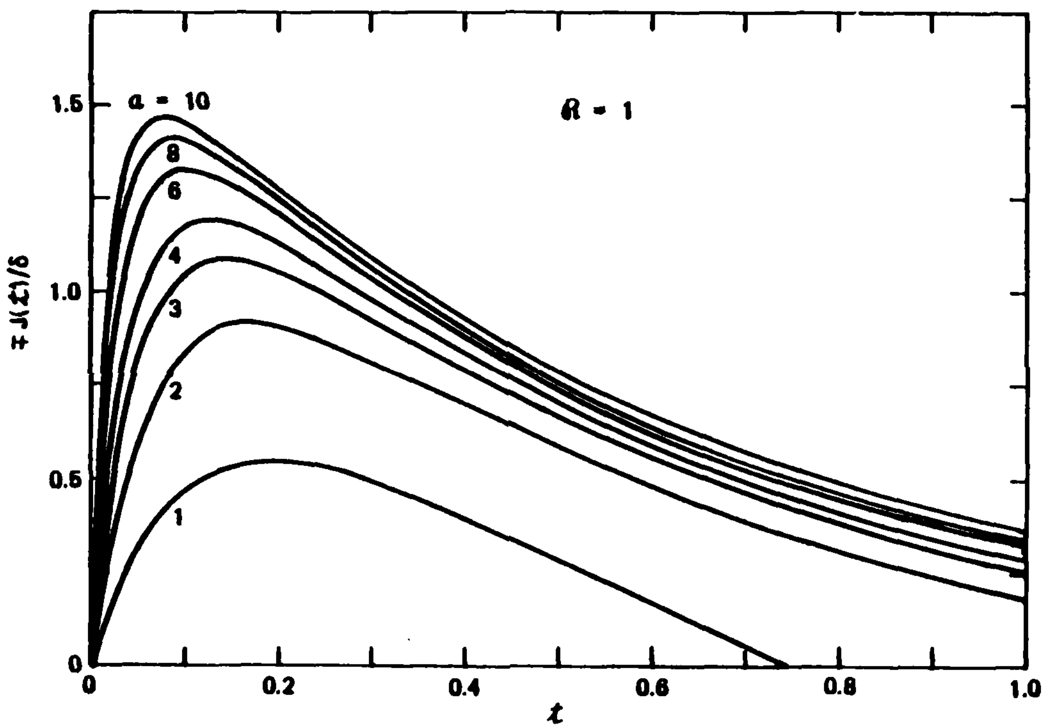

FIG. 4: Current pulse $\mp$ $J(t)/\delta$ ("$\pm$" polarization) versus time t for $R = 1$, and $a = 1$ to 10.

CONCLUSION

A one-dimensional theory of a solid state power generator has been presented, in which the electromotoric force is produced by electric polarization of a solid in the stress field of a shock wave. The numerical results indicate that energies of the order $\Delta E \sim 10^5$ Joule can possibly be generated per pulse. This type of power generator does not require an external magnetic field and appears to be promising as a pulsed power source for special applications (for which the efficiency of transformation of chemical energy of explosives into stress waves and electrical energy is of no concern).

Experiments should be carried through to verify the theoretical expectations. In comparing experimental and theoretical current pulse forms, it should be noted that the temporal current distribution depends noticeably on the shape of the stress wave. Current pulse forms for other stress wave shapes than the simple step wave can be calculated in complete analogy. It might also be of interest to refine the theory by including multi-dimensional effects and deformation of the solid.

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INITIAL-BOUNDARY-VALUE PROBLEM FOR DIFFUSION OF MAGNETIC FIELDS INTO CONDUCTORS WITH EXTERNAL ELECTROMAGNETIC TRANSIENTS

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ABSTRACT

The initial-boundary-value problem for the diffusion of an initially homogeneous magnetic field into a slab of conductivity $\sigma < \infty$ and width $\Delta x = 2a$ is solved, under consideration of the electromagnetic wave pulses generated at the surfaces of the conductor by its interaction with the external magnetic field, which propagate into the surrounding vacuum. The analytical solutions show that (i) the external electromagnetic transients are necessary in order to correctly satisfy the boundary conditions for the tangential electric and magnetic field components, and (ii) the spatial and temporal development of the electromagnetic field and electric current in the conductor is quantitatively determined by a new dimensionless parameter group $R = \mu_o \sigma a c$ $[c = (\mu_o \varepsilon_o)^{-1/2}]$. This "magnetic Reynolds number of the vacuum" determines the coupling between the transient fields in the conductor $\sigma > 0$ and the ambient space ($\sigma = 0$).

INTRODUCTION

The diffusion of electromagnetic fields $\vec{B}(\vec{r},t)$, $\vec{E}(\vec{r},t)$ in a conductor of finite conductivity σ and normal surface vector $\vec{n}(\vec{s})$, when the electromagnetic field $\vec{B}_o(\vec{r},t)$ and $\vec{E}_o(\vec{r},t)$ outside of the conductor are known, is in general described by Maxwell's equations without displacement current, where the tangential field components are assumed to satisfy the boundary conditions[1,2] $\vec{n} \times [\vec{B}(\vec{r},t) - \vec{B}_o(\vec{r},t)] = \vec{0}$ and $\vec{n} \times [\vec{E}(\vec{r},t) - \vec{E}_o(\vec{r},t)] = \vec{0}$. If the external electromagnetic field is time-independent and electric potential fields are absent, then $\vec{B}_o = \vec{B}_o(\vec{r})$ and $\vec{E}_o = \vec{0}$ (since $\nabla \times \vec{E}_o = -\partial\vec{B}_o/\partial t = \vec{0}$ and $\vec{E}_o = -\nabla\Phi_o \equiv 0$), so that the tangential boundary conditions reduce to[1,2] $\vec{n} \times [\vec{B}(\vec{r},t) - \vec{B}_o(\vec{r})] = \vec{0}$ and $\vec{n} \times \vec{E}(\vec{r},t) = \vec{0}$. These boundary conditions have found widespread use in mathematical physics[1], electromagnetic theory[2], and the theory of magnetic flux compression (at the outside surface of the liners)[3,4]. However, these boundary conditions are questionable approximations, since they do not take into consideration the wave fields $\vec{B}(\vec{r},t)$, $\vec{E}(\vec{r},t)$ propagating away from the conductor into the surrounding medium, which have their sources in the transient current fields $\vec{j} = \mu_o^{-1} \nabla \times \vec{B}$ of the conductor.

For a concrete illustration of the problematics, consider the diffusion of an external (homogeneous) magnetic field, $\vec{B}_o = \{0,B_o,0\}$ for $|x| > a$, into a conducting slab in the region $|x| < a$ which is field free at time $t = 0$ (Fig. 1). Using the conventional boundary conditions, the transient magnetic field $\vec{B}(x,t) = \{0,B(x,t),0\}$ in the conductor is determined by the parabolic initial-boundary- value problem:[5]

$$\partial B/\partial t = \kappa\partial^2 B/\partial x^2 \quad , \quad |x| < a \quad , \quad t > 0 \quad , \qquad (1)$$

$$B(x,t=0) = 0 \quad , \quad |x| < a \quad , \qquad (2)$$

$$B(x=\pm a,t) = B_o \quad , \quad t > 0 \quad , \qquad (3)$$

where $\kappa = 1/\mu_o\sigma$. By means of Fourier's method, the general solution of Eqs. (1) - (3) is obtained as a superposition of eigenfunctions[5]:

$$B(x,t) = B_o[1 - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)} e^{-\kappa(2n-1)^2\pi^2 t/4a^2} \cos\frac{2n-1}{2a}\pi x], \quad |x| < a, \quad t > 0, \quad (4)$$

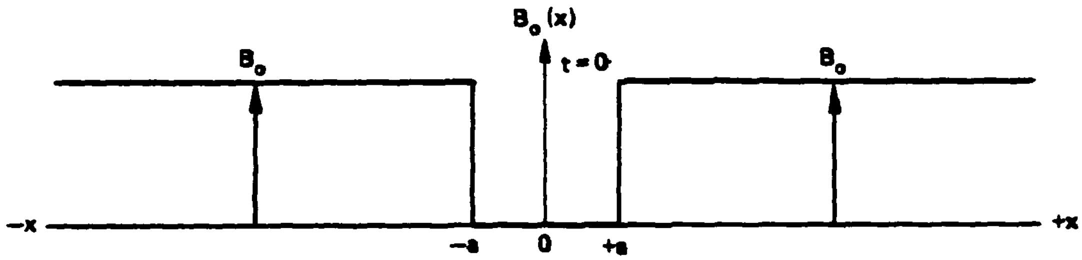

FIG. 1: Magnetic Field $B_o(x)$ for $t = 0$.

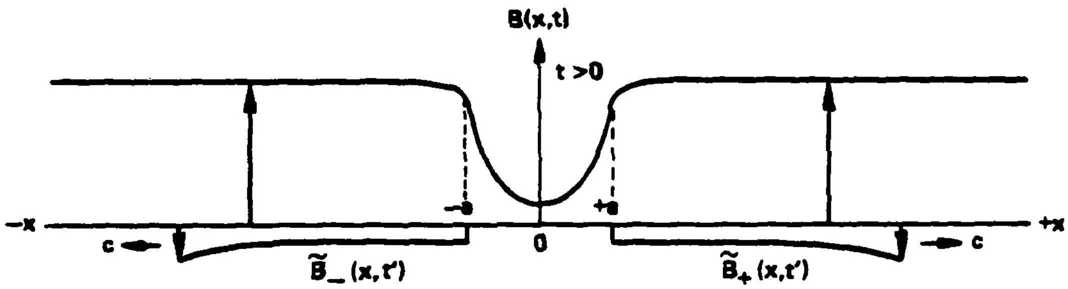

FIG. 2: Diffusion Field $B(x,t)$ and
Transients $\widetilde{B}_{\pm}(x,t)$ for $t > 0$
and $t' > 0$ (qualitative).

with $B(x,t) \to B_o$ in $|x| < a$ for $t \to \infty$. Since $\nabla \times \vec{B} = \mu_o \sigma \vec{E}$, the electric field $\vec{E}(x,t) = \{0,0,E(x,t)\}$ in the conductor is

$$E(x,t) = \frac{2B_o}{\mu_o \sigma a} \sum_{n=1}^{\infty} (-1)^{n-1} e^{-\kappa(2n-1)^2 \pi^2 t/4a^2} \sin \frac{2n-1}{2a} \pi x \quad , \quad |x| < a,\ t > 0. \qquad (5)$$

In accordance with the boundary conditions (3), the space outside of the conductor remains unperturbed while the electromagnetic field diffuses into the conductor,

$$B_o(x,t) = B_o \quad , \quad E_o(x,t) = 0 \quad , \quad |x| > a \quad , \quad t > 0 \quad . \qquad (6)$$

The transient currents $\vec{j} = \nabla \times \vec{B}/\mu_o$ in the conductor are "eddy currents," and, therefore, cannot produce transient magnetic fields $\tilde{B}_o(x,t) = B_o(x,t) - B_o \neq 0$ in the outside region $|x| > a$. The net current $I(t)$ through any cross section z = constant vanishes, due to the boundary conditions (3):

$$I(t)/\Delta y = \mu_o^{-1} \int_{-a}^{+a} [\partial B(x,t)/\partial x] dx = \mu_o^{-1} \int_{B_o}^{B_o} dB = 0 \quad . \qquad (7)$$

By comparing the conductor solutions (4) and (5) with the vacuum solutions (6), it is seen that $B(x = \pm a,t) - B_o = 0$, but $E(x = \pm a,t) - E_o(x = \pm a,t) = E(x = \pm a,t) \neq 0$! Thus, the conventional boundary conditions[1,2] lead to a violation of the fundamental law of the continuity of the tangential electric fields at interfaces.

The correct formulation of the boundary conditions requires consideration of the simultaneous wave fields $\tilde{B}_\pm(x,t)$, $\tilde{E}_\pm(x,t)$ propagating with the speed of light c into the positive and negative half-spaces $x > +a$ and $x < -a$ surrounding the conductor (Fig. 2), which are excited by the transient current fields $j(x,t) = \mu_o^{-1} \partial B(x,t)/\partial x$ in the (space-charge free) conductor. No matter how small the external transients $\tilde{B}_\pm(x,t)$ and $\tilde{E}_\pm(x,t)$ are (in comparison with $B_o \neq 0$ and $E_o = 0$), they have to be taken into account in order to rigorously satisfy the boundary conditions $\vec{n} \times [\vec{B}] = \vec{0}$ and $\vec{n} \times [\vec{E}] = \vec{0}$ for the continuity of the tangential electromagnetic fields at conductor interfaces.

The quantitative assessment of the significance of the external transients of the diffusion process leads to the discovery of a new dimensionless parameter combination, which has the physical meaning of a "magnetic Reynolds number of the vacuum":

$$R = \mu_o \sigma a c \quad , \quad c = (\mu_o \varepsilon_o)^{-1/2} = 3 \times 10^8 \text{ m/sec} \quad . \tag{8}$$

In the following, the formulation of the initial-boundary-value problem for diffusion processes with external transients and its analytical solutions for the transient electromagnetic fields inside and outside the conductor are presented. The qualitative and quantitative importance of the new boundary conditions and the external wave fields are discussed in terms of R.

The presented theory has important implications for the evaluation of the flux losses through the liners of magnetic field compressors[3,4], the electromagnetic acceleration of conducting macroparticles[6,7], the electromagnetic induction in conductors moving relative to external magnetic fields[8,9], and for the interaction of transient plasma shock waves with external magnetic fields[10,11]. The general significance for theoretical physics is obvious.

INITIAL-BOUNDARY-VALUE PROBLEM

Subject of the consideration is the diffusion of the magnetic field into a conducting slab $|x| < a$, which is initially embedded in a homogeneous magnetic field $\vec{B}_o = \{0,B_o,0,\}$, under simultaneous emission of electromagnetic waves from the conductor surfaces $x = \pm a$ (Fig. 1).

The transient electromagnetic fields $\vec{B}_\pm = \{0,B_\pm(x,t),0\}$ and $\vec{E}_\pm = \{0,0,E_\pm(x,t)\}$ in the infinite vacuum half-spaces ($\sigma = 0$, $\mu = \mu_o$) $x > +a$ and $x < -a$ are determined by the initial-boundary-value problems ($\pm$) for the wave equation,

$$\partial^2 B_\pm/\partial t^2 = c^2 \partial^2 B_\pm/\partial x^2, \quad \pm x > a, \quad t > 0, \tag{9}$$

$$B_\pm(x,t = 0) = B_o \quad , \quad \pm x > a \quad , \tag{10}$$

$$B_\pm(x = \pm a,t) = B_o + \psi_\pm(t) \quad , \quad t > 0, \tag{11}$$

since

$$\partial E_\pm/\partial t = c^2 \partial B_\pm/\partial x \quad , \quad \partial E_\pm/\partial x = \partial B_\pm/\partial t \tag{12}$$

by Maxwell's equations with displacement current. The solutions of Eqs. (9) - (11) for the still undetermined boundary values $\psi_\pm(t)$ are:

$$\begin{aligned} B_\pm(x,t) &= B_o + \psi_\pm(t \mp \frac{x \mp a}{c}) \quad , \quad a < \pm x < a + ct \quad , \\ &= B_o \quad , \quad a + ct < \pm x < \infty \quad , \end{aligned} \tag{13}$$

and

$$\begin{aligned} E_\pm(x,t) &= \mp c\psi_\pm(t \mp \frac{x \mp a}{c}) \quad , \quad a < \pm x < a + ct \quad , \\ &= 0 \quad , \quad a + ct < \pm x < \infty \quad . \end{aligned} \tag{14}$$

These solutions are typical for hyperbolic equations, i.e., the boundary values $\psi_\pm(t)$ are "transported" into the half-spaces $\pm x > a$ with the speed of light c, so that discontinuous wave fronts result at $x = \pm(a + ct)$.

Let the external magnetic field $\vec{B}_o$ be switched on at $t = 0$ so that no electromagnetic fields exist in the conductor for $t < 0$[1,2]. The conductor has a finite conductivity σ and can, therefore, not carry surface current densities,[12] $\vec{j}^* = \lim_{\Delta x \to 0} \sigma \vec{E} \Delta x = 0$ for $\sigma < \infty$ and $\vec{E}$ bounded. Accordingly, the boundary conditions for the tangential electric and magnetic field components at the conductor vacuum interfaces are

$$B(x = \pm a, t) = B_o + \psi_{\pm}(t) \quad , \quad t > 0 \quad , \tag{15}$$

$$E(x = \pm a, t) = \mp c\psi_{\pm}(t) \quad , \quad t > 0 \quad , \tag{16}$$

where

$$E(x,t) = \kappa \partial B(x,t)/\partial x \quad , \quad |x| < a \quad , \quad t > 0 \quad , \tag{17}$$

by Ohm's law is the electric field in the conductor, and $B(x,t)$ is the magnetic field in $|x| < a$. Furthermore

$$\kappa = 1/\mu_o \sigma > 0 \quad . \tag{18}$$

(The boundary conditions $\vec{n} \cdot [\varepsilon \vec{E}] = 0$ and $n \cdot [\vec{B}] = \vec{0}$ are satisfied since $\vec{E}$ and $\vec{B}$ have no normal components.) By elimination of the unknown boundary values $E(x = \pm a, t)$ and $\psi_{\pm}(t)$ from Eqs. (15) - (17), boundary conditions involving only the magnetic field $B(x,t)$ in the conductor are obtained:

$$\frac{\partial B(x=\pm a,t)}{\partial x} \pm \frac{c}{\kappa} B(x = \pm a, t) = \pm \frac{c}{\kappa} B_o \quad , \quad t > 0 \quad . \tag{19}$$

These are the new boundary conditions for the diffusion of magnetic fields $B(x,t)$ into conductors. They differ from the conventional boundary conditions[1,2] through the curl terms $\partial B(x = \pm a, t)/\partial x \neq 0$, which consider the emission of magnetic dilution waves from the conductor surfaces $x = \pm a$ into the vacuum $|x| > a$.

Within the conducting slab of finite width 2a, the propagation of the magnetic field can be treated in the nonrelativistic or diffusion approximation[1,2,12]. Accordingly, $B(x,t)$ in the initially field free conductor is determined by the parabolic initial-boundary-value problem:

$$\partial B/\partial t = \kappa \partial^2 B/\partial x^2 \quad , \quad |x| < a \quad , \quad t > 0 \quad , \tag{20}$$

$$B(x,t = 0) = 0 \quad , \quad |x| < a \quad , \tag{21}$$

$$\partial B(x = \pm a,t)/\partial x \pm hB(x=\pm a,t) = \pm hB_o , \quad t > 0 \quad , \tag{22}$$

where

$$h = c/\kappa > 0 \quad . \tag{23}$$

The transformation,

$$B(x,t) = B_o + \tilde{B}(x,t) \quad , \quad |x| \leq a \quad , \quad t \geq 0 \quad , \tag{24}$$

reduces Eqs. (21) - (22) to an initial-boundary-value problem with standard "radiation" boundary conditions:

$$\partial \tilde{B}/\partial t = \kappa \partial^2 \tilde{B}/\partial x^2 \quad , \quad |x| < a \quad , \quad t > 0 \quad , \tag{25}$$

$$\tilde{B}(x,t = 0) = -B_o \quad , \quad |x| < a \quad , \tag{26}$$

$$\partial \tilde{B}(x = \pm a,t)/\partial x \pm h\tilde{B}(x=\pm a,t) = 0 \quad . \tag{27}$$

In accordance with Fourier's theorem, the solution of Eqs. (25) - (27) is obtained as a superposition of eigen-functions $\tilde{B}_n(x,t)$ for the region $|x| \leq a$ which satisfy the boundary conditions (27):

$$\tilde{B}(x,t) = -2B_o \sum_{n=1}^{\infty} \frac{(h^2+k_n^2)k_n^{-1}\sin k_n a}{[(h^2+k_n^2)a+h]} e^{-\kappa k_n^2 t} \cos k_n x \quad , \quad |x| \leq a \quad , \quad t \geq 0 \quad , \tag{28}$$

where

$$k_n a \ \mathrm{tg}(k_n a) = ha \quad , \quad n = 1,2,3,\ldots, \tag{29}$$

gives the eigenvalues k_n associated with the boundary conditions (27).

1. <u>Conductor Solutions</u>:

For a compact representation of the field solutions, dimensionless independent (ξ,τ) and dependent variables are introduced,

$$\xi = x/a \quad , \quad \tau = \kappa t/a^2 \quad , \quad \alpha_n = k_n a \quad . \tag{30}$$

$$\mathcal{B}(\xi,\tau) = B(x,t)/B_o, \quad \mathcal{E}(\xi,\tau) = E(x,t)/(\kappa B_o/a), \quad j(\xi,\tau) = j(x,t)/(B_o/\mu_o a). \tag{31}$$

According to Eqs. (24) and (28), the solutions for the dimensionless fields $\mathcal{B}(\xi,\tau)$, $\mathcal{E}(\xi,\tau) = \partial\mathcal{B}(\xi,\tau)/\partial\xi$, and $j(\xi,\tau)$ in the conductor are given by:

$$\mathcal{B}(\xi,\tau) = 1 - 2\sum_{n=1}^{\infty} \frac{(R^2+\alpha_n^2)\alpha_n^{-1}\sin\alpha_n}{[(R^2+\alpha_n^2)+R]} e^{-\alpha_n^2\tau}\cos\alpha_n\xi \; , \; |\xi| \leq 1 \; , \; \tau \geq 0 \; , \tag{32}$$

$$\mathcal{E}(\xi,\tau) = 2\sum_{n=1}^{\infty} \frac{(R^2+\alpha_n^2)\sin\alpha_n}{[(R^2+\alpha_n^2)+R]} e^{-\alpha_n^2\tau}\sin\alpha_n\xi \qquad , \; |\xi| \leq 1 \; , \; \tau \geq 0 \; , \tag{33}$$

$$j(\xi,\tau) = \mathcal{E}(\xi,\tau) \; , \; |\xi| \leq 1 \; , \; \tau \geq 0 \; , \tag{34}$$

where

$$\alpha_n \mathrm{tg}\alpha_n = R \; , \; n = 1,2,3,\ldots, \; R = ac/\kappa = \mu_o\sigma ac \tag{35}$$

by Eqs. (8) and (29). For sufficiently large times $\tau >> \alpha_1^{-2}$, the homogeneous magnetic field has diffused completely into the conductor,

$$\mathcal{B}(\xi,\tau) \to 1 \; , \; \mathcal{E}(\xi,\tau) \to 0 \; , \; j(\xi,\tau) \to 0 \; , \; \tau \to \infty \quad . \tag{36}$$

In the hypothetical limit of infinite magnetic vacuum Reynolds number R, Eq. (32) reduces to the known solution (4) for the conventional boundary conditions[5],

$$\lim_{R\to\infty}\mathcal{B}(\xi,\tau) = 1 - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)} e^{-(2n-1)^2\pi^2\tau/4}\cos\frac{2n-1}{2}\pi\xi, \; |\xi| \leq 1, \; \tau \geq 0, \tag{37}$$

since

$$\lim_{R\to\infty} \alpha_n = \frac{2n-1}{2}\pi \; , \quad n = 1,2,3,\ldots \quad . \tag{38}$$

Comparison of Eq. (32) with Eq. (37) indicates that the $\mathcal{B}(\xi,\tau)$ solutions with the new and conventional boundary conditions differ not much if $R >> \alpha_1 = \pi/2$.

2. Vacuum Solutions:

In view of the boundary conditions (15), Eq. (32) yields for the boundary values $\Psi_{\pm}(\tau) = B(\xi=\pm 1,\tau) - 1$. Accordingly, Eqs. (13) and (14) give for the electromagnetic fields in the positive ($\xi > +1$) and negative ($\xi < -1$) half-spaces the solutions:

$$B_{\pm}(\xi,\tau) = 1 + \Psi_{\pm}(\tau \mp \frac{\xi\mp 1}{R}) \;, \quad 1 < \pm\xi < 1 + R\tau \;, \tag{39}$$
$$= 1 \;, \quad 1 + R\tau < \pm\xi < \infty \;,$$

and

$$E_{\pm}(\xi,\tau) = \mp R\Psi(\tau \mp \frac{\xi\mp 1}{R}) \;, \quad 1 < \pm\xi < 1 + R\tau \;, \tag{40}$$
$$= 0 \;, \quad 1 + R\tau < \pm\xi < \infty \;,$$

where

$$\Psi_{\pm}(\tau \mp \frac{\xi\mp 1}{R}) = -2\sum_{n=1}^{\infty} \frac{(R^2+\alpha_n^2)\sin\alpha_n\cos\alpha_n}{\alpha_n[(R^2+\alpha_n^2)+R]} e^{-\alpha_n^2[\tau\mp(\xi\mp 1)/R]} \;. \tag{41}$$

Eqs. (39) and (40) represent electromagnetic wave pulses which propagate with the dimensionless speed R(c) from the conductor surfaces $\xi = \pm 1$ into the vacuum spaces $\pm\xi > 1$ with discontinuous wave fronts at $\xi = \pm(1 + R\tau)$. They are kicked on at $\tau = 0$ and their emission lasts to the end ($\tau \to \infty$) of the diffusion process in the conductor. The vacuum fields $B_{\pm}(\xi,\tau)$ are in opposite direction of $\vec{B}_o$, i.e., they represent dilution waves (Fig. 2).

In the case of large coupling numbers, $R \gg 1$, Eqs. (39) and (40) yield with Eq. (41)

$$B_{\pm}(\xi,\tau) \doteq 1 + O[R^{-1}] \;, \quad 1 < \pm\xi < 1 + R\tau \;, \quad \tau > 0, \quad R \gg 1, \tag{42}$$

$$E_{\pm}(\xi,\tau) \doteq \pm 2\sum_{n=1}^{\infty} e^{-(\frac{2n-1}{2}\pi)^2\tau} \;, \quad 1 < \pm\xi < 1 + R\tau \;, \quad \tau > 0, \quad R \gg 1, \tag{43}$$

since $\cos\alpha_n \doteq (-1)^{n-1}(2n-1)\pi/2R$ for $R \gg n\pi$ by Eq. (35).

The magnetic field $\vec{B}_0$ outside of the conductor remains nearly unperturbed during the diffusion process, $\tilde{B}_\pm \sim R^{-1}$, whereas the external electric transients $E_\pm \neq 0$ are of order R^0 behind the wave fronts, $\xi = \pm(1 + R\tau)$, for $R >> 1$. However, since $\nabla\times\vec{B}_\pm = c^{-2}\partial\vec{E}_\pm/\partial t$, not only $E_\pm \sim R^0$ but also $\tilde{B}_\pm \sim R^{-1}$ cannot be neglected for $R >> 1$.

Thus, we have shown how self-consistent solutions can be obtained for the electric and magnetic fields in the conductor and the surrounding vacuum, which satisfy the boundary conditions for the continuity of the tangential electric and magnetic fields at the conductor-vacuum interfaces. The conventional boundary conditions for electromagnetic diffusion processes[1,2], ignore the external electromagnetic transients, violate the boundary condition for the tangential electric field, and permit no Poynting vector $\vec{S} = \vec{E}\times\vec{H}$ outside of the conductor. As a result, the conventional boundary conditions [1,2] make it impossible for electromagnetic fields to diffuse through conductors and to escape into the ambient space.

For both the conductor and vacuum solutions, the limit $R \to 0$, which implies $\sigma \to 0$ since $a \neq 0$, is not realizable since the conductivity of conductors is by definition large. For insulators or extremely poor conductors ($R \propto \sigma \to 0$), the nonrelativistic or parabolic diffusion equation is inapplicable[12]. Therefore, the investigation of the limit $R \to 0$ would require solution of Maxwell's equations with displacement current in the slab $|x| < a$ given elsewhere[13].

The generalization of the theory required for conductors and external media (vacuum, gases, fluids) with different permittivities ε and μ are trivial but complicate the notation.

DISCUSSION

It is known that Maxwell's equations with displacement current and the non-relativistic Ohm's law $\vec{j} = \sigma\vec{E}$ combine to a hyperbolic diffusion equation for the magnetic field $\vec{B}(\vec{r},t)$ in conductors[12],

$$\frac{\partial^2\vec{B}}{\partial t^2} + \frac{1}{\tau_R}\frac{\partial\vec{B}}{\partial t} = c^2\nabla^2\vec{B} \tag{44}$$

where

$$\tau_R = \varepsilon_o/\sigma \tag{45}$$

is the field relaxation time, e.g., $\tau_R \cong (10^{-9}/36\pi)/6\times10^7 \sim 10^{-19}$sec for copper with $\sigma = 6\times10^7\Omega^{-1}/\text{m}$. Eq. (44) reduces to the parabolic diffusion equation in the limit $\tau_R << 1$:

$$\frac{\partial\vec{B}}{\partial t} = \tau_R c^2\nabla^2\vec{B} \quad , \quad \tau_R << 1 \quad . \tag{46}$$

The parabolic diffusion equation is an excellent approximation, since the relaxation time of conductors is extremely small, $\tau_R << 1$. By Eqs. (45) and (46), the field relaxation time τ_R and the diffusion time τ_D are interrelated by

$$\tau_D^{-1} = \tau_R c^2/a^2 \quad , \quad \tau_D = \mu_o\sigma a^2 \quad , \tag{47}$$

where a is the extension of the conductor. For conductors, the diffusion time is relatively large if a is not microscopically small, e.g., $\tau_D = 4\pi\times10^{-7}6\times10^7\times10^{-4} \sim 10^{-2}$sec for a copper slab of width $a = 10^{-2}$m.

Comparison of the neglected term $\partial^2\vec{B}/\partial t^2$ with the second and third terms of Eq. (44) reveals the relation of the parabolic diffusion approximation to the new coupling number $R = \mu_o\sigma ac$:

$$\left|\frac{\partial^2\vec{B}}{\partial t^2}\right| / \left|\tau_R^{-1}\frac{\partial\vec{B}}{\partial t}\right| \sim \left|\frac{\partial^2\vec{B}}{\partial t^2}\right| / \left|c^2\nabla^2\vec{B}\right| \sim \frac{\tau_R}{\tau_D} = \frac{\varepsilon_o\mu_o}{(\mu_o\sigma a)^2} = R^{-2} \quad . \tag{48}$$

This result again confirms the validity of the parabolic diffusion equation for conductors, for which $R = \mu_o\sigma ac >> 1$. E.g., $R \doteq 4\pi\times10^{-7}6\times10^7\times10^{-2}3\times10^8 \sim 10^8$ for a copper slab $a = 10^{-2}$m. More important, Eq. (48) demonstrates that the neglected relativistic term $\partial^2\vec{B}/\partial t^2$ in the conductor is small of order $R^{-2} <<<< 1$, whereas the calculated electromagnetic fields in the conductor are of order $B \sim E \sim R^0$ [Eqs. (32) - (33)], and the external electromagnetic transients are of order $\tilde{B}_\pm \sim R^{-1}$ and $E_\pm \sim R^0$ [Eqs. (39) - (40)], since in Eq. (41) for large R

$$|\cos\alpha_n| = [1 + (R/\alpha_n)^2]^{-1/2} \doteq \alpha_n/R \sim \frac{2n-1}{2}\pi R^{-1} \;, \; R >> n\pi \;, \; n = 1,2,3,\ldots \; . \tag{49}$$

In conclusion, it is noted that, in conductors, magnetic field diffusion is a nonrelativistic process (as is electric conduction, $\vec{j} = \sigma\vec{E}$). The electric transients $E_\pm$ in vacuum must be of the same order as the electric field E in the conductors, $E_\pm \sim E \sim R^0$, since otherwise the tangential electric field could not be continuous across the conductor-vacuum interface. On the other hand, the external magnetic transients $\tilde{B}_\pm$ are small of order $R^{-1} = (\mu_o\sigma a)^{-1}c^{-1}$ since the magnetic field energy flows with the speed of light towards the conducting cavity. The deeper physical reason for these electromagnetic transients is to be seen in the conservation laws for electromagnetic energy and momentum, which follow from Maxwell's equations[12)].

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INITIAL-BOUNDARY-VALUE PROBLEM FOR ELECTROMAGNETIC INDUCTION IN ACCELERATED CONDUCTORS MOVING ACROSS MAGNETIC FIELDS

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Abstract

Boundary conditions are derived for the interfaces of a conductor moving across an external magnetic field in an ambient medium (vacuum or non-conductor), which consider the emission of electromagnetic waves from the conductor surface as a result of electromagnetic induction. These boundary conditions are applied to the initial-boundary-value problem for the electromagnetic induction in a conducting slab, which is accelerated across a homogeneous magnetic field to a non-relativistic velocity. Fourier series solutions are presented for the transient electromagnetic fields in the moving conductor and the discontinuous electromagnetic waves in the ambient space. It is shown that the transient electromagnetic fields inside and outside the conductor are due to two mechanisms, i.e., "velocity induction" (ordinary induction) and "acceleration induction" ($d\vec{v}(t)/dt \neq \vec{0}$).

INTRODUCTION

Although the theoretical foundations for electromagnetic induction in conductors moving across magnetic fields were formulated in 1908 by H. Minkowski[1], only simple problems such as stationary unipolar induction in rotating discs have been discussed.[2] The electrodynamics of moving media has been a subject of basic research and also controversy to date.[3] In the treatment of electromagnetic induction, e.g., in liners of magnetic field compressors[4,5] and transient plasma shock waves interacting with external magnetic fields $\vec{B}_o$[6,7], it has become customary to use the boundary condition for the tangential magnetic field in the form $\vec{n}\times[\vec{B} - \vec{B}_o] = \vec{0}$ at the conductor-gas interface, where $\vec{B}$ is the transient magnetic field in the conductor and $\vec{B}_o$ is the unperturbed (!) external magnetic field. This boundary condition leads to electromagnetic solutions $\vec{B},\vec{E}$ in the conductor which do not satisfy the corresponding boundary condition $\vec{n}\times[\vec{E} - \vec{E}_o] = \vec{0}$ for the tangential electric fields at the moving interface. These "conventional" boundary conditions produce approximate to incorrect results, depending on the physical situation.

For an illustration of the problematics of the conventional boundary conditions,[4,5] which are also being used in the analysis of magnetic field diffusion into conductors at rest[8], consider a conducting slab $\Delta x = 2a$ with its surfaces initially at $x = \pm a$ in a transverse homogeneous magnetic field $\vec{B}_o = \{0,B_o,0\}$ for $|x| < \infty$ (Fig. 1). At time $t = 0$, this conductor is set into motion with a velocity $\vec{v}(t) = \{d\hat{x}(t)/dt,0,0\}$ so that its front and rear surfaces are at $x = x(t) \pm a$ at time $t > 0$ where $\hat{x}(t=0) = 0$. No matter whether the induction of the transient magnetic field $\vec{B}(x,t)$ in the moving conductor

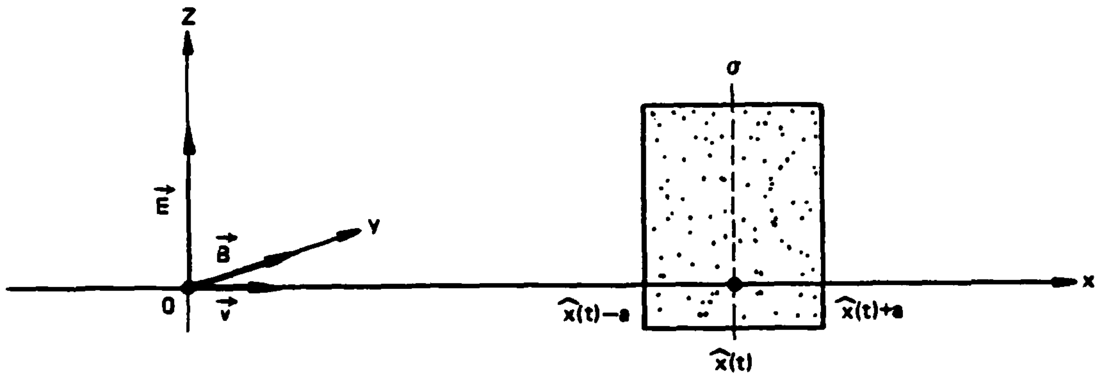

FIG. 1: Geometry of fields $\vec{B}$, $\vec{v}$, $\vec{E}$, and location $\hat{x}(t)$ of conductor for $t \geq 0$.

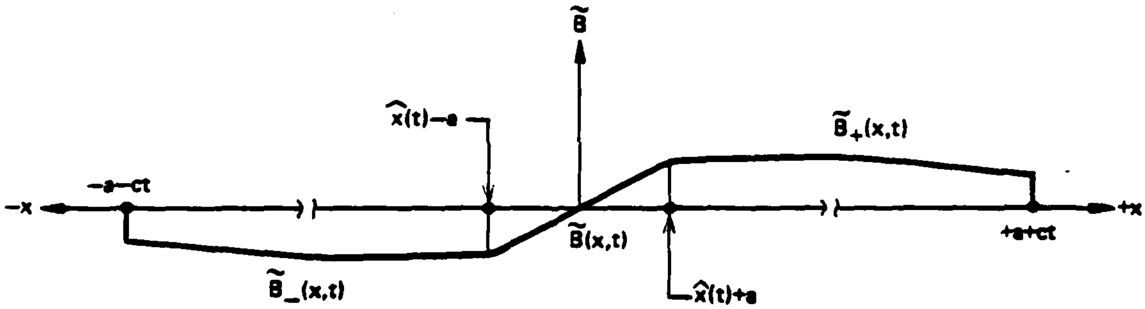

FIG. 2: Qualitative representation of induced field $\tilde{B}(x,t)$ in moving conductor and external electromagnetic waves $\tilde{B}_{\pm}(x,t)$ with fronts at $x = \pm(a + ct)$.

is described by the relativistic wave equation[9] or the non-relativistic diffusion equation[9], the initial condition $\vec{B}(x,t=0) = \vec{B}_o$, $-a < x < +a$, and the conventional boundary conditions $\vec{B}(x=\hat{x}(t)\pm a,t) = \vec{B}_o$, $t > 0$, permit only one and the same solution, $\vec{B}(x,t) = \vec{B}_o$, which implies $\vec{E}(x,t) = -\vec{v}(t)x\vec{B}_o$, and $\vec{J}(x,t) = \vec{0}$ by Ohm's law for the moving conductor. These simple solutions are due to the conventional boundary conditions without external perturbations and are obviously not correct, since the boundary condition for the tangential electric field is not satisfied, $\vec{n}x[\vec{E} - \vec{E}_o] = v(t)B_o\vec{e}_y \neq \vec{0}$ where $\vec{E}_o = \vec{0}$. Since M. Faraday it is an experimental fact that transient electromagnetic fields and currents are induced in the conducting slab as soon as it is moved relative to the external magnetic field $\vec{B}_o$.

In the following, an analytical solution is presented for the initial-boundary-value problem of the electromagnetic induction in a conducting slab $\Delta x = 2a$, which is at rest for $t < 0$ in a transverse, homogeneous magnetic field $\vec{B}_o$, and which is set in motion at $t = 0$ with a nonrelativistic velocity $\vec{v}(t)$ of arbitrary (finite) acceleration $d\vec{v}(t)/dt$ (Figs. 1,2). The electromagnetic induction in the conductor is shown to produce transient electromagnetic fields within the conductor and electromagnetic waves at the moving conductor surfaces $x = \hat{x}(t) \pm a$, which propagate with the speed of light in the surrounding space to infinity. Only if the electromagnetic waves outside of the moving conductor are taken into consideration, consistent solutions of Maxwell's equations exist which satisfy the boundary conditions for the tangential and normal electromagnetic fields at the conductor-non-conductor interfaces.

The analytical solutions for the moving conductor and surrounding spaces permit significant conclusions. The transient electric field induced in the conductor is the sum of a field which is proportional to the velocity $\vec{v}(t)$ and a field which is an integral functional of the acceleration $d\vec{v}(t)/dt$ of the conductor. A fundamental dimensionless group (σ = conductivity, μ_1 = permeability of conductor)

$$R = \mu_1 \sigma a c, \quad c = (\mu_2 \varepsilon_2)^{-1/2}$$

is found which represents a "magnetic Reynolds number" of (i) "free space" (if the conductor moves in vacuum or gas with permeabilities ε_o, μ_o,), or (ii) "non-conducting space" (if the conductor moves in a nonconducting medium with permeabilities ε_2, μ_2). R determines the coupling of the transient electromagnetic fields inside and outside the conductor. The external magnetic transients would be negligible for $R \to \infty$, however, the external electric transients do not vanish for $R \to \infty$. Thus, the conventional boundary conditions which assume that the external magnetic field $\vec{B}_o$ remains unperturbed outside of the conductor while transient electromagnetic processes occur in the latter are not correct within electromagnetic theory.

BOUNDARY CONDITIONS

For the analysis of electromagnetic induction in moving conductors (conductivity $\sigma < \infty$, permittivities ε_1,μ_1), which move with a velocity field $\vec{v}(\vec{r},t)$ relative to the "laboratory system" in a non-conducting medium (vacuum, gas, or fluid; $\sigma = 0$, ε_2,μ_2), the boundary conditions for the electromagnetic fields $\vec{B}_1$, $\vec{E}_1$ (conductor) and $\vec{B}_2$, $\vec{E}_2$ (non-conductor) are required in the L-frame. Integration of Maxwell's equations with displacement current[10)] over the interface 1-2 with velocity $\vec{v}(\vec{r},t)$ and normal vector $\vec{n}(\vec{r},t)$ (direction $1 \to 2$) yields for the tangential and normal electromagnetic field components the boundary conditions in the L-system:

$$\vec{n}\times[\vec{E}_2 - \vec{E}_1] = +(\vec{n}\cdot\vec{v})[\vec{B}_2 - \vec{B}_1] \quad , \tag{1}$$

$$\vec{n}\times[\vec{B}_2/\mu_2 - \vec{B}_1/\mu_1] = \vec{j}^* - (\vec{n}\cdot\vec{v})[\varepsilon_2\vec{E}_2 - \varepsilon_1\vec{E}_1] \quad , \tag{2}$$

$$\vec{n}\cdot[\varepsilon_2\vec{E}_2 - \varepsilon_1\vec{E}_1] = \rho^* \quad , \tag{3}$$

$$\vec{n}\cdot[\vec{B}_2 - \vec{B}_1] = 0 \quad , \tag{4}$$

where

$$\vec{j}^* = \rho^*\vec{v} \quad , \quad \rho^* = \lim_{\substack{\Delta s\to 0\\ |\rho|\to\infty}} \rho\Delta s \quad , \tag{5}$$

are the surface current and charge densities of the interface $\Delta s \to 0$. A conductive surface current density does not exist at a conductor of finite conductivity, $\vec{j}^*_\sigma = \lim_{\Delta s\to 0} \sigma\vec{E}\Delta s = \vec{0}$ for $\sigma < \infty$ and $\vec{E}$ bounded.

In many cases, the permittivities of good conductors and their ambient atmospheres equal the free space values, $\varepsilon_{1,2} = \varepsilon$ and $\mu_{1,2} = \mu$. With this simplification in notation, the boundary conditions (1) - (4) are applied to the front (+) and rear (-) surfaces $x = \hat{x}(t) \pm a$ of a conducting slab ($\sigma < \infty$) with the fields (Fig. 1)

$$\vec{B} = \{0,B(x,t),0\} \quad , \quad \vec{E} = \{0,0,E(x,t)\} \quad , \quad \hat{x}(t) - a < x < \hat{x}(t)+a \quad ;$$

$$\vec{B}(x,t=0) = \vec{B}_o \quad , \quad \vec{E}(x,t=0) = \vec{0} \quad , \quad -a < x < +a, \quad , \tag{6}$$

which moves with the velocity $\vec{v}(t) = \{d\hat{x}(t)/dt,0,0\}$ in an ambient medium ($\sigma = 0$) with the fields (Fig. 1)

$$\vec{B}_\pm = \{0,B_\pm(x,t),0\} \quad , \quad \vec{E}_\pm = \{0,0,E_\pm(x,t)\}, \quad x \gtrless x(t) \pm a \quad ;$$

$$\vec{B}_\pm(x,t=0) = \vec{B}_o \quad , \quad \vec{E}_\pm(x,t=0) = \vec{0} \quad , \quad x \gtrless \pm a \quad . \tag{7}$$

$\vec{B}_o$ is an external homogeneous magnetic field which fills uniformly the conductor (1) and the medium (2). The boundary conditions (3) and (4) are satisfied identically since $\vec{B},\vec{E}$ and $\vec{B}_\pm,\vec{E}_\pm$ are $\perp$ $\vec{n}$ so that $\vec{j}^* = \vec{0}$ and $\rho^* = 0$ by Eq. (5). The tangential boundary conditions (1) and (2) yield for the fields (6) - (7):

$$E_\pm(x=\hat{x}(t)\pm a,t) - E(x=\hat{x}(t)\pm a,t) = \mp v(t)[B_\pm(x=\hat{x}(t)\pm a,t) - B(x=\hat{x}(t)\pm a,t)], \tag{8}$$

$$B_\pm(x=\hat{x}(t)\pm a,t) - B(x=\hat{x}(t)\pm a,t) = \mp v(t)c^{-2}[E_\pm(x=\hat{x}(t)\pm a,t) - E(x=\hat{x}(t)\pm a,t)], \tag{9}$$

where

$$c = (\mu\varepsilon)^{-1/2} \tag{10}$$

is the speed of light. For non-relativistic conductor motions, Eqs. (8) - (9) reduce to:

$$E_\pm(x=\hat{x}(t)\pm a,t) - E(x=\hat{x}(t)\pm a,t) = 0 \quad , \quad v(t)^2 << c^2 \quad , \tag{11}$$

$$B_\pm(x=\hat{x}(t)\pm a,t) - B(x=\hat{x}(t)\pm a,t) = 0 \quad , \quad v(t)^2 << c^2 \quad . \tag{12}$$

According to the non-relativistic Ohm's law for conductors which are moving with a velocity $\vec{v}$ relative to the L-system, $\vec{j} = \sigma(\vec{E} + \vec{v}\times\vec{B})$, the electric field $\vec{E}(x,t)$ in the conductor is expressed in terms of $\vec{B}(x,t)$,

$$E = -vB + (\mu\sigma)^{-1}\partial B/\partial x \quad , \quad \hat{x}(t) - a < x < \hat{x}(t) + a \;,\; v(t)^2 << c^2 \quad . \qquad (13)$$

The electromagnetic field $\vec{B}_\pm(x,t)$, $\vec{E}_\pm(x,t)$ in the adjacent semi-infinite half spaces is determined by the hyperbolic initial-boundary-value problems:

$$\partial^2 B_\pm/\partial t^2 = c^2 \partial^2 B_\pm/\partial x^2 \quad , \quad x \gtrless \hat{x}(t) \pm a \quad , \quad t > 0 \quad , \qquad (14)$$

$$B_\pm(x,t=0) = B_o \qquad , \quad x \gtrless \pm a \qquad , \qquad (15)$$

$$B_\pm(x=\hat{x}(t)\pm a,t) = B(x=\hat{x}(t)\pm a,t), \qquad t > 0 \quad , \qquad (16)$$

since

$$\partial E_\pm/\partial x = \partial B_\pm/\partial t \qquad , \quad \partial B_\pm/\partial x = c^{-2}\partial E_\pm/\partial t \qquad , \qquad (17)$$

by Maxwell's equations for homogeneous non-conductors. Eq. (16) couples the solutions $B_\pm(x,t)$ in the semi-infinite spaces $x \gtrless \hat{x}(t) \pm a$ to the solution $B(x,t)$ in the conductor, $\hat{x}(t) - a < x < \hat{x}(t) + a$. By Eqs. (14) - (17), the ambient electromagnetic field transients are of the form:

$$\begin{aligned} B_\pm(x,t) &= B_o + \Psi_\pm\left(t \mp \frac{x \mp a}{c}\right) \quad , \quad \hat{x}(t) \pm a \lesssim x \lesssim \pm(a + ct) \quad , \\ &= B_o \qquad\qquad\qquad\qquad , \quad a + ct < \pm x < \infty \qquad , \qquad (18) \end{aligned}$$

and

$$\begin{aligned} E_\pm(x,t) &= \mp c\Psi_\pm\left(t \mp \frac{x \mp a}{c}\right) \quad , \quad \hat{x}(t) \pm a \lesssim x \lesssim \pm(a + ct) \quad , \\ &= 0 \qquad\qquad\qquad\qquad , \quad a + ct < \pm x < \infty \qquad , \qquad (19) \end{aligned}$$

where

$$\Psi_\pm\left(t \mp \frac{\hat{x}(t)}{c}\right) = B(x=\hat{x}(t)\pm a,t) - B_o \qquad (20)$$

by Eq. (16) determines the form of the wave functions $\Psi_\pm(\eta_\pm)$ of the "self similar" arguments $\eta_\pm = t \mp x/c + a/c$ from the conductor solution $B(x,t)$.

The solutions (18) - (19) are typical for hyperbolic initial-boundary-value problems, i.e., the boundary values $\Psi_\pm(t \mp \hat{x}(t)/c)$ are transported with the speed of light c into the half spaces $x \gtrless \hat{x}(t) \pm a$, so that discontinuous wave fronts result at $x = \pm(a + ct)$.

By means of Eqs. (18) - (20), the boundary-values $B_\pm(x{=}\hat{x}(t)\pm a,t)$, $E_\pm(x{=}\hat{x}(t)\pm a,t)$, and $\Psi_\pm(t \mp \hat{x}(t)/c)$ are eliminated from the boundary conditions (11) and (12):

$$\frac{\partial B(x{=}\hat{x}(t)\pm a,t)}{\mu\sigma\partial x} \pm [c \mp v(t)]B(x{=}\hat{x}(t)\pm a,t) = \pm cB_o, \quad v(t)^2 << c^2 \quad . \qquad (21)$$

This are the fundamental new boundary conditions for moving conductors which (i) involve only boundary values of the magnetic field B(x,t) of the conductor, and (ii) consider the emission of electromagnetic waves,

$$\tilde{B}_\pm(x,t) = \Psi_\pm(t \mp \frac{x \mp a}{c}) \quad , \quad \tilde{E}_\pm(x,t) = \mp c\Psi(t \mp \frac{x \mp a}{c}), \qquad (22)$$

from the conductor surfaces $x = \hat{x}(t) \pm a$ into the ambient spaces $x \gtrless x(t) \pm a$. Since the magnetic field in the conductor is the sum of the external B_o and a transient $\tilde{B}(x,t)$,

$$B(x,t) = B_o + \tilde{B}(x,t) \ , \ \hat{x}(t) - a < x < \hat{x}(t) + a \quad , \qquad (23)$$

Eq. (21) gives for the transient conductor field the boundary conditions:

$$\frac{\partial \tilde{B}(x{=}\hat{x}(t)\pm a,t)}{\mu\sigma\partial x} \pm [c \mp v(t)]\tilde{B}(x{=}\hat{x}(t)\pm a,t) = v(t)B_o, \quad v(t)^2 << c^2 \quad . \qquad (24)$$

In view of the difficulties to accelerate macroscopic bodies to speeds $|v| > 10^4$m sec^{-1}, the stronger non-relativistic condition $|v| << c$ is in general fulfilled, which reduces Eqs. (21) and (24) to the simpler boundary conditions:

$$(\mu\sigma)^{-1} \frac{\partial B(x{=}\hat{x}(t)\pm a,t)}{\partial x} \pm c[B(x{=}\hat{x}(t)\pm a,t) - B_o] = v(t)B_o \ , \quad |v(t)| << c \ , \qquad (25)$$

and

$$(\mu\sigma)^{-1} \frac{\partial \tilde{B}(x=\hat{x}(t)\pm a,t)}{\partial x} \pm c\tilde{B}(x=\hat{x}(t)\pm a,t) = v(t)B_o \quad , \quad |v(t)| << c \quad . \tag{26}$$

If the $\nabla\times\vec{B}$ and $v(t)B_o$ terms are omitted, Eqs. (25) and (26) reduce to the conventional boundary conditions[4-7] $B(x=\hat{x}(t)\pm a,t) = B_o$ and $\tilde{B}(x=\hat{x}(t)\pm a,t) = 0$, respectively. Comparison shows that the conductor currents $\nabla\times\vec{B}/\mu$ and the induced currents $\sigma v(t)B_o$ at the conductor surface are the sources of the emitted electromagnetic transients.

INITIAL-BOUNDARY-VALUE PROBLEM

Consider a rigid conducting slab of width $\Delta x = 2a$ with surfaces in the planes $x = \hat{x}(t) \pm a$ at time $t \geqslant 0$ (Fig. 1). This conductor is exposed to an external magnetic field $\vec{B}_o = \{0, B_o, 0\}$ which is homogeneous throughout the space $-\infty < x < +\infty$, and is accelerated to a (non-relativistic) velocity $\vec{v}(t) = \{v(t), 0, 0\}$ from an initial position at rest $\hat{x}(t=0) = 0$ so that

$$v(t) = d\hat{x}(t)/dt \quad , \quad \hat{x}(t) = \int_0^t v(t')dt' \quad , \quad dv(t)/dt \gtrless 0 \quad , \quad (27)$$

with

$$\hat{x}(t=0) = 0 \quad , \quad v(t=0) = 0 \quad , \quad dv(t=0)/dt \gtrless 0 \quad . \quad (28)$$

The electromagnetic induction of the transient electromagnetic fields $\vec{B} = \{0, B(x,t), 0\}$, $\vec{E} = \{0, 0, E(x,t)\}$ in the conductor of finite width, as a result of its accelerated motion $\vec{v}(t)$ across the external magnetic field $\vec{B}_o$, is determined by the parabolic initial-boundary-value problem for $B(x,t)$:

$$\frac{\partial B}{\partial t} + v(t)\frac{\partial B}{\partial x} = \kappa\frac{\partial^2 B}{\partial x^2} \quad , \quad \hat{x}(t) - a < x < \hat{x}(t) + a \quad , \quad t > 0 \quad , \quad (29)$$

$$B(x, t=0) = B_o \quad , \quad -a < x < +a \quad , \quad (30)$$

$$\frac{\partial B(x=\hat{x}(t) \pm a, t)}{\partial x} \pm h[B(x=\hat{x}(t) \pm a, t) - B_o] = \kappa^{-1} v(t) B_o \quad , \quad t > 0 \quad . \quad (31)$$

where

$$\kappa = 1/\mu\sigma \quad , \quad h = c/\kappa \quad . \quad (32)$$

Eq. (29) follows from Maxwell's equations and Ohm's law $\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$ for moving conductors in the diffusion approximation,[9] $\tau_D = \epsilon/\sigma \to 0$. The boundary conditions (31) couple the electromagnetic induction process in the conductor to the external transients in the ambient medium [Eq. (25)]. The transformation,

$$B(x,t) = B_o + \tilde{B}(x,t) \quad , \quad -a \leqslant x \leqslant +a \quad , \quad t \geqslant 0 \quad , \quad (33)$$

$$\chi = x - \hat{x}(t) \qquad , \quad t \geq 0 \qquad , \quad (34)$$

reduces Eqs. (29) - (31) to the initial-boundary-value problem:

$$\partial\tilde{B}/\partial t = \kappa\partial^2\tilde{B}/\partial\chi^2 \quad , \quad -a < \chi < +a \quad , \quad t > 0 \qquad , \quad (35)$$

$$\tilde{B}(\chi,t=0) = 0 \qquad , \quad -a < \chi < +a \quad , \qquad (36)$$

$$\partial B(\chi=\pm a,t)/\partial\chi \pm h\tilde{B}(\chi=\pm a,t) = \kappa^{-1}v(t)B_o \quad , \quad t > 0 \qquad , \quad (37)$$

The linear IBVP (35) - (37) is decomposed into a BVP and an IBVP by means of the ansatz:

$$\tilde{B}(\chi,t) = F(\chi,t) + G(\chi,t) \quad , \quad -a \leq \chi \leq +a \quad , \quad t \geq 0 \qquad , \quad (38)$$

where

$$\partial^2 F/\partial\chi^2 = 0 \quad , \quad -a < \chi < +a \quad , \quad t > 0 \qquad , \quad (39)$$

$$\partial F(\chi=\pm a,t)/\partial\chi \pm hF(\chi=\pm a,t) = \kappa^{-1}v(t)B_o \quad , \quad t > 0 \qquad , \quad (40)$$

and

$$\partial G/\partial t = \kappa\partial^2 G/\partial\chi^2 - \partial F/\partial t \quad , \quad -a < \chi < +a \quad , \quad t > 0 \qquad , \quad (41)$$

$$G(\chi,t=0) = -F(\chi,t=0) \quad , \quad -a < \chi < +a \qquad , \quad (42)$$

$$\partial G(\chi=\pm a,t)/\partial\chi \pm hG(\chi=\pm a,t) = 0 \quad , \quad t > 0 \qquad . \quad (43)$$

The solution $F(\chi,t)$ of Eqs. (29) - (30), the source $\partial F(\chi,t)/\partial t$ in Eq. (41), and the initial value $F(\chi,t=0)$ in Eq. (42) are:

$$F(\chi,t) = [v(t)B_o/2(\kappa + ac)]\chi \quad , \quad -a \leq \chi \leq +a \quad , \quad t \geq 0 \qquad , \quad (44)$$

$$\partial F(\chi,t)/\partial t = [\frac{dv(t)}{dt} B_o/2(\kappa + ac)]\chi \quad , \quad -a \leq \chi \leq +a \quad , \quad t \geq 0 \qquad , \quad (45)$$

and

$$F(\chi,t=0) = 0 \qquad , \quad -a \leq \chi \leq +a \qquad (46)$$

since $v(t=0) = 0$. With $\partial F(x,t)/\partial t$ odd in x and $F(x,t=0) = 0$, the initial-boundary-value problem (41) - (43) is solved by means of the Fourier expansions:

$$G(x,t) = \sum_{n=1}^{\infty} G_n(t)\sin k_n x \quad , \quad -a \leq x \leq +a \quad , \quad t \geq 0 \quad , \tag{47}$$

$$\partial F(x,t)/\partial t = \sum_{n=1}^{\infty} S_n(t)\sin k_n x \ , \ -a \leq x \leq +a \quad , \quad t \geq 0 \quad , \tag{48}$$

where

$$k_n a \ \mathrm{ctg}\ k_n a = -ha \quad , \quad n = 1,2,3, \ldots \quad , \tag{49}$$

determines the eigen-values k_n of the eigen-functions $\sin k_n x$ associated with the boundary conditions (43). Substitution of Eqs. (47) - (48) into Eqs. (41) and (42) yields by means of the inverse Fourier theorem

$$dG_n(t)/dt + \kappa k_n^2 G_n(t) = -S_n(t) \quad , \quad t > 0 \quad , \tag{50}$$

$$G_n(t=0) = 0 \quad , \tag{51}$$

where

$$S_n(t) = (a/\kappa)K_n B_o dv(t)/dt \quad , \quad t \geq 0 \quad , \tag{52}$$

$$K_n = (h^2 + k_n^2)\sin k_n a/(k_n a)^2[(h^2 + k_n^2) + (h/a)] \quad . \tag{53}$$

by Eqs. (45) and (48). The solution of Eqs. (50) and (51) is

$$G_n(t) = -\int_0^t e^{-\kappa k_n^2(t-t')} S_n(t')dt' \quad , \quad t \geq 0 \quad . \tag{54}$$

Combining of Eqs. (47), (52), and (54) yields as solution of the initial-boundary-value problem (41) - (43)

$$G(x,t) = -\frac{a}{\kappa}B_o \sum_{n=1}^{\infty} K_n \left[\int_0^t \frac{dv(t')}{dt'} e^{-\kappa k_n^2(t-t')} dt'\right]\sin k_n x, \ -a \leq x \leq +a, \ t \geq 0 \ . \tag{55}$$

By Eqs. (33), (34), (38), (44), and (55), the magnetic field in the moving conductor is

$$B(x,t) = B_o + [v(t)B_o/2(\kappa+ac)](x-\hat{x}(t)) - \frac{a}{\kappa}B_o\sum_{n=1}^{\infty}K_n[\int_0^t \frac{dv(t')}{dt'}e^{-\kappa k_n^2(t-t')}dt']\sin k_n(x-\hat{x}(t)),$$

$$\hat{x}(t) - a \leq x \leq \hat{x}(t) + a \quad , \quad t \geq 0 \qquad . \qquad (56)$$

This is a fundamental result, which shows that the transient magnetic field $\tilde{B}(x,t)$ is the sum of a field $F(x,t)$ induced by the motion $v(t)$ and a field $G(x,t)$ induced by the acceleration $dv(t)/dt$ of the conductor in the external magnetic field B_o. Similar decompositions exist for the electric field $E(x,t)$ and current density $j(x,t)$ in the accelerated conductor.

The still unknown forms ($\pm$) of the two external wave functions $\Psi_\pm(t \mp x/c + a/c)$ in the spaces $x \gtrless \hat{x}(t) \pm a$ are determined from the solution (56) by means of the boundary condition (20), which gives

$$\Psi_\pm(t \mp \frac{\hat{x}(t)}{c}) = \tilde{B}(x=\pm a,t) \quad , \quad t \geq 0 \qquad . \qquad (57)$$

The transformations $t_\pm = t_\pm(t)$ and its inverses $t = f_\pm\{t_\pm\}$ for the two ($\pm$) waves defined by

$$t_\pm = t \mp \frac{\hat{x}(t)}{c} \quad , \quad t = f_\pm\{t_\pm\} \quad , \quad t \geq 0 \qquad , \qquad (58)$$

where $t_+ = t_- = 0$ for $t = 0$ ($\hat{x}(t=0) = 0$) but $t_+ \neq t_-$ for $t > 0$, show that the wave functions are of the form $\Psi_\pm(t_\pm) = \tilde{B}(x=\pm a, t = f_\pm\{t_\pm\})$. Accordingly,

$$\Psi_\pm(t \mp \frac{x \mp a}{c}) = \tilde{B}(x=\pm a, t=f_\pm\{t \mp \frac{x \mp a}{c}\}), \quad \hat{x}(t) \pm a \lesseqgtr x \lesseqgtr \pm(a + ct) \ , \ t \geq 0. \qquad (59)$$

Since $\tilde{B}(x,t) = B(x,t) - B_o$, substitution of Eq.(56) into Eq. (59) gives the wave functions $\Psi_\pm$ as functionals of $f_\pm\{t \mp x/c + a/c\}$:

$$\Psi_\pm\left(t \mp \frac{x \mp a}{c}\right) = \pm[aB_o/2(\kappa + ac)]v\left(f_\pm\left\{t \mp \frac{x \mp a}{c}\right\}\right)$$

$$\mp \frac{a}{\kappa} B_o \sum_{n=1}^{\infty} K_n \sin k_n a \, e^{-\kappa k_n^2 f_\pm\{t \mp \frac{x \mp a}{c}\}} \int_0^{f_\pm\{t \mp \frac{x \mp a}{c}\}} \frac{dv(t')}{dt'} e^{\kappa k_n^2 t'} dt' ,$$

$$\hat{x}(t) \pm a \lesssim x \lesssim \pm(a + ct) \quad , \quad t > 0 \qquad . \qquad (60)$$

The solutions (60) determine the propagation of the emitted electromagnetic waves outside of the moving conductor, $x \gtrless \hat{x}(t) \pm a$. Again, these waves consist each of a "velocity" wave [v(t)] and an "acceleration" wave [dv(t)/dt]. They satisfy all boundary and initial conditions,

$$\Psi_\pm/_{x = \hat{x}(t) \pm a} = \tilde{B}(x=\pm a,t) \quad , \quad t > 0 \qquad , \qquad (61)$$

$$\Psi_\pm/_{t = 0} = 0 \qquad , \quad x \gtrless \pm a \qquad , \qquad (62)$$

since $t_\pm = 0$ and $x = \pm a$ [Eq. (60)] for $t = 0$, and, hence, $f_\pm = f_\pm(0) = 0$ [Eq. (58)] and $v(f_\pm) = v(0) = 0$.

For a brief illustration of the transformations (58) consider the simple conductor motion $v(t) = v_o$, $\hat{x}(t) = v_o t$, $t > 0$. In this case, $t_\pm = (1 \mp v_o/c)t$ and $t = t_\pm/(1 \mp \frac{v_o}{c})$, i.e., $f_\pm(t_\pm)$ is proportional to t .

ANALYTICAL SOLUTIONS

For the most general representation of the electromagnetic fields in the moving conductor and the ambient medium, dimensionless independent and dependent variables are introduced by

$$\xi = x/a,\ \tau = \kappa t/a^2,\ \hat{\xi}(\tau) = \hat{x}(t)/a,\ \alpha_n = k_n a,\ \nu(\tau) = v(t)/v_o \tag{63}$$

$$\mathcal{B}(\xi,\tau) = B(x,t)/B_o,\quad \mathcal{E}(\xi,\tau) = E(x,t)/(\kappa B_o/a),\quad j(\xi,\tau) = j(x,t)/(B_o/\mu a),$$

$$\psi(\xi,\tau) = \Psi(x,t)/B_o \quad . \tag{64}$$

1. Conductor Solutions

According to Eq. (56), the dimensionless electromagnetic fields $\mathcal{B}(\xi,\tau)$, $j(\xi,\tau) = \partial\mathcal{B}(\xi,\tau)/\partial\xi$, and $\mathcal{E}(\xi,\tau)$ in the accelerated conductor are:

$$\mathcal{B}(\xi,\tau) = 1 + \frac{M}{2(1+R)}\nu(\tau)[\xi - \hat{\xi}(\tau)]$$

$$- M\sum_{n=1}^{\infty} K_n\left[\int_0^{\tau} \frac{d\nu(\tau')}{d\tau'} e^{-\alpha_n^2(\tau-\tau')} d\tau'\right]\sin\alpha_n[\xi - \hat{\xi}(\tau)] \quad ,$$

$$\hat{\xi}(\tau) - 1 < \xi < \hat{\xi}(\tau) + 1 \ ,\quad \tau > 0 \quad , \tag{65}$$

$$j(\xi,\tau) = \frac{M}{2(1+R)}\nu(\tau)$$

$$- M\sum_{n=1}^{\infty} \alpha_n K_n\left[\int_0^{\tau} \frac{d\nu(\tau')}{d\tau'} e^{-\alpha_n^2(\tau-\tau')} d\tau'\right]\cos\alpha_n[\xi - \hat{\xi}(\tau)] \quad ,$$

$$\hat{\xi}(\tau) - 1 < \xi < \hat{\xi}(\tau) + 1 \ ,\quad \tau > 0 \quad , \tag{66}$$

$$\mathcal{E}(\xi,\tau) = -M\nu(\tau)\mathcal{B}(\xi,\tau) + j(\xi,\tau),\ \hat{\xi}(\tau) - 1 < \xi < \hat{\xi}(\tau) + 1,\ \tau > 0 \quad , \tag{67}$$

where

$$\alpha_n \mathrm{ctg}\alpha_n = - R \quad , \quad n = 1,2,3, \ldots \qquad , \qquad (68)$$

$$K_n = (R^2 + \alpha_n^2)\sin\alpha_n/\alpha_n^2[(R^2 + \alpha_n^2) + R] \qquad , \qquad (69)$$

and

$$M = \mu\sigma a v_o \qquad , \qquad (70)$$

$$R = \mu\sigma a c \qquad . \qquad (71)$$

M is known as the magnetic Reynolds number of the conductor with characteristic speed $\sim v_o$. Eqs. (65) - (67) indicate that $M/(1 + R)$ determines the order of the ratio $\tilde{B}/B_o$ of induced and external magnetic fields. The steady-state induction in moving conductors [11,12] is determined only by M.

R is a new dimensionless group which involves the velocity of light $c = (\varepsilon\mu)^{-1/2}$ so that $R \gg M$ for $v_o \ll c$. R has the physical meaning of a "magnetic Reynolds number" of the ambient nonconducting space, i.e., R is a coupling parameter between the conductor ($0 < \sigma < \infty$) and its external medium ($\sigma = 0$), which determines the magnitudes of the external electromagnetic transients [Eqs. (75) - (76)].

The net electric current flowing through the conductor is per unit width $\Delta\eta = 1$

$$I(\tau) = \int_{\hat{\xi}(\tau)-1}^{\hat{\xi}(\tau)+1} j(\xi,\tau)d\xi = M\nu(\tau)/(1 + R)$$
$$- 2M \sum_{n=1}^{\infty} \sin\alpha_n K_n \int_0^{\tau} \frac{d\nu(\tau')}{d\tau'} e^{-\alpha_n^2(\tau-\tau')} d\tau' \quad , \quad \tau > 0 \qquad . \qquad (72)$$

2. <u>External Solutions</u>

For the space surrounding the moving conductor, the wave functions $\Psi_\pm(x,t)$ of the electromagnetic transients are by Eqs. (60),(63) - (64) in dimensionless representation

$$\psi_{\pm}(\tau \mp \frac{\xi \mp 1}{R}) = \pm\frac{M}{2(1+R)}\,\nu(\phi_{\pm}\{\tau \mp \frac{\xi \mp 1}{R}\})$$

$$\mp M \sum_{n=1}^{\infty} \sin\alpha_n K_n e^{-\alpha_n^2 \phi_{\pm}\{\tau \mp \frac{\xi \mp 1}{R}\}} \int_0^{\phi_{\pm}\{\tau \mp \frac{\xi \mp 1}{R}\}} \frac{d\nu(\tau')}{d\tau'} e^{\alpha_n^2 \tau'} d\tau' \quad ,$$

$$\hat{\xi}(\tau) \pm 1 \leq \xi \leq \pm(1 + R\tau) \ , \ \tau > 0, \quad (73)$$

where

$$\phi_{\pm}\{\tau \mp \frac{\xi \mp 1}{R}\} \equiv (\kappa/a^2) f_{\pm}\{t \mp \frac{x \mp a}{c}\} \quad . \quad (74)$$

By Eqs. (18) - (19), and (73) the dimensionless solutions for the electromagnetic field outside the moving conductor are:

$$B_{\pm}(\xi,\tau) = 1 + \psi_{\pm}(\tau \mp \frac{\xi \mp 1}{R}) \quad , \quad \hat{\xi}(\tau) \pm 1 \leq \xi \leq \pm(1 + R\tau), \quad \tau > 0 \ ,$$

$$= 1 \quad , \quad 1 + R\tau < \pm\xi < \infty \quad , \quad \tau > 0 \quad (75)$$

and

$$E_{\pm}(\xi,\tau) = \mp R\psi_{\pm}(\tau \mp \frac{\xi \mp 1}{R}) \quad , \quad \hat{\xi}(\tau) \pm 1 \leq \xi \leq \pm(1+R\tau) \ , \quad \tau > 0 \ ,$$

$$= 0 \quad , \quad 1 + R\tau < \pm\xi < \infty \quad , \quad \tau > 0 \ . \quad (76)$$

DISCUSSION

The magnetic Reynolds number is $M = \mu\sigma a v_o \lesssim 1$ for conductors (depending on the parameters σ, a, and v_o), whereas the coupling number $R = \mu_o \sigma a c >> 1$ is large for macroscopic (a) conductors. In the most general case, electromagnetic induction in a moving conductor is determined both by M and R, where $M << R$ since $v_o << c$.

1. Case $R >> 1$. For most macroscopic conductors, it is $\sigma > 10^4 \Omega^{-1}/m$ and $a > 10^{-4} m$ ($\mu = 4\pi\times10^{-7}$ Vsec/Am, $c = 3\times10^8$ m/sec) so that $R > 10^2$, and by Eqs. (68) and (69)

$$\alpha_n \doteq n\pi \quad , \qquad R >> n\pi \, , \quad n = 1,2,3, \ldots \quad , \tag{77}$$

$$K_n \doteq (-1)^{n+1}(n\pi)^{-1}R^{-1} \quad , \qquad R >> n\pi \, , \quad n = 1,2,3, \ldots \quad , \tag{78}$$

It is seen that the "velocity" and "acceleration" fields in Eqs. (65) - (67) are of the same order since

$$M/(1 + R) \sim M(+K_1) \sim M/R = v_o/c << 1 \, , \quad R >> 1 \quad . \tag{79}$$

In the magnetic field solution (65), the dominant term is the external $B_o = 1 >> M/R$, and in the electric field solution (67) the dominant term is the motion induced field $|M\nu B_o| >> |j|$, if $R >> 1$. Although significant electric fields are induced in the conductor, the induced magnetic field is small, $\tilde{B} \sim M/(1 + R) << B_o = 1$ if $R >> 1$, and, therefore, the current density $j = \partial B/\partial\xi$ is small, too.

2. Case $R = \infty$. In an actual experiment, $R = \infty$ can never be reached but only asymptotically approached. In this hypothetical situation, the conductor [Eqs. (65) - (67)] and external [Eqs. (75) - (76)] solutions reduce to:

$$B(\xi,\tau) = 1 \, , \; j(\xi,\tau) = 0, \; E(\xi,\tau) = -M\nu(\tau), \; \hat{\xi}(\tau) - 1 < \xi < \hat{\xi}(\tau) + 1, \; \tau > 0, \tag{80}$$

and

$$B(\xi,\tau) = 1 + O[MR^{-1}], \; E_\pm(\xi,\tau) \sim MR^0 \neq 0 \quad , \quad \hat{\xi}(\tau) \pm 1 \lesssim \xi \lesssim \pm(1+R\tau) \; , \; \tau > 0 \quad , \tag{81}$$

since

$$\lim_{R\to\infty} \alpha_n = n\pi \;, \quad \lim_{R\to\infty} K_n = 0 \;, \quad \lim_{R\to\infty} RK_n = (-1)^{n+1}/n\pi \,, \quad n = 1,2,3,\ldots \,. \tag{82}$$

It should be noted that for $R \to \infty$, only $\tilde{B}_\pm = 0$ but $E_\pm \neq 0$, i.e., the external electric transients are (behind their wave fronts) of the same order of magnitude as the electric field E in the conductor . Since $\nabla\times\vec{B}_\pm = c^{-2}\partial\vec{E}_\pm/\partial t$, the electric transients $E_\pm$ always coexist with (no matter how small) magnetic transients $\tilde{B}_\pm$. For this reason, the limit $R = \infty$ has no physical meaning, quite apart from the fact that always $R < \infty$ for $\sigma, a, c < \infty$. The conventional boundary conditions[4,5,8] for electromagnetic diffusion processes in conductors imply $R = \infty$ and $E_\pm \equiv 0$, and are, therefore, physically incorrect.

As an explanation it is noted that, in conductors, magnetic field diffusion is a nonrelativistic process, as is electric conduction, $\vec{j} = \sigma(\vec{E} + \vec{v}\times\vec{B})$. The electric transients $E_\pm$ in vacuum must be of the same order as the electric field E in the conductors, $E_\pm \sim E \sim MR^0$, since otherwise the tangential electric field would not be continuous across the conductor surface. On the other hand, the external magnetic transients $\tilde{B}_\pm$ are small of order $MR^{-1} = M(\mu_0\sigma a)^{-1}c^{-1}$ since the magnetic field energy flows with the speed of light in the ambient space. The deeper physical reason for the external electromagnetic transients is to be seen in the conservation laws for electromagnetic energy and momentum, which follow from Maxwell's equations[9].

3. Diffusion Approximation. It is known that Maxwell's equations with displacement current and the nonrelativistic Ohm's law, $\vec{j} = \sigma(\vec{E} + \vec{v}\times\vec{B})$, combine to a hyperbolic diffusion equation for the magnetic field $\vec{B}$ in conductors[9], which reads in the considered one-dimensional case with $\vec{r}$-independent conductor velocity v(t)

$$\frac{\partial^2 B}{\partial t^2} + \frac{1}{\tau_R}\frac{\partial B}{\partial t} + \frac{v}{\tau_R}\frac{\partial B}{\partial x} = c^2\,\frac{\partial^2 B}{\partial x^2} \tag{83}$$

where

$$\tau_R = \varepsilon_o/\sigma << 1 \tag{84}$$

is the field relaxation time, which is extremely small for conductors. Eq. (83) reduces to the parabolic diffusion equation in the limit $\tau_R << 1$:

$$\frac{\partial B}{\partial t} + v\frac{\partial B}{\partial x} = \tau_R c^2 \frac{\partial^2 B}{\partial x^2} \quad , \quad \tau_R << 1 \quad . \tag{85}$$

The parabolic diffusion equation is an excellent approximation, since the relaxation time of conductors is very small, $\tau_R << 1$. By Eqs. (84) and (85), the field relaxation time τ_R and the diffusion time τ_D are interrelated by

$$\tau_D^{-1} = \tau_R c^2/a^2 \quad , \quad \tau_D = \mu_o \sigma a^2 \quad , \tag{86}$$

where a is the extension of the conductor. For conductors, the diffusion time is relatively large if a is not too small, i.e., $\tau_D >> \tau_R$.

Comparison of the neglected term $\partial^2 B/\partial t^2$ with the leading $(0 \leq |v| << c)$ second and fourth terms of Eq. (83) reveals the relation of the parabolic diffusion approximation to the new coupling number $R = \mu_o \sigma ac$:

$$\left|\frac{\partial^2 B}{\partial t^2}\right| / \left|\tau_R^{-1} \frac{\partial B}{\partial t}\right| \sim \left|\frac{\partial^2 B}{\partial t^2}\right| / \left|c^2 \frac{\partial^2 B}{\partial x^2}\right| \sim \frac{\tau_R}{\tau_D} = \frac{\varepsilon_o \mu_o}{(\mu_o \sigma a)^2} = R^{-2} \quad . \tag{87}$$

This result again confirms the validity of the parabolic diffusion equation for conductors, for which $R = \mu_o \sigma ac >> 1$. More important, Eq. (87) demonstrates that the neglected relativistic term $\partial^2 B/\partial t^2$ is small of order $R^{-2} <<<< 1$, whereas the calculated electromagnetic fields in the conductor are of order $B \sim E \sim MR^0$ [Eqs. (65) - (67)], and the external electromagnetic transients are of order $\tilde{B}_\pm \sim MR^{-1}$ and $E_\pm \sim MR^0$ [Eqs. (73) - (76)].

Thus, consistent electromagnetic field solutions for the regions in and outside of the accelerated conductor have been obtained within the parabolic diffusion approximation, which satisfy the boundary conditions for the continuity of the tangential electric and magnetic fields. If an accuracy of higher order R^{-2} is to be achieved, then the hyperbolic diffusion equation (83) has to be used. For non-relativistic conductor motions, $|v| << c$, however, an accuracy of order R^{-1} is completely sufficient. The mathematical advantages of the parabolic diffusion equation become obvious if it is used in connection with (time-dependent) moving boundary conditions (magnetic flux compressors, electromagnetic induction generators, etc.), which are extremely difficult to treat for the hyperbolic diffusion equation.

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QUANTUM-KINETIC THEORY OF ELECTRON HEATING IN PLASMAS BY HIGH-FREQUENCY ELECTROMAGNETIC WAVES

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ABSTRACT

The heating rate of electrons in laser-irradiated plasmas is derived from the quantum-mechanically extended Vlasov equation. The heating of electrons is shown to be due to (i) the electron velocity space diffusion by the plasma waves, which are induced by the laser field (classical turbulent heating), (ii) the microscopic quantum-mechanical interaction between each electron and the laser field in the electric field of the plasma waves (collisionless inverse bremsstrahlung), and (iii) collisional inverse bremsstrahlung in the static plasma field. A quasilinear equation for the changes of the electron distribution function by these three concomitant mechanisms is derived and the heating rates of the electrons are calculated. It is shown that the collisionless inverse bremsstrahlung is dominant for hot plasmas in most experimental cases. The so-called anomalous heating is identified as being due to collisionless inverse bremsstrahlung, and is explicitly expressed in terms of the wave number of the plasma wave and the plasma parameters, when the turbulence is due to the ion acoustic instability and is stabilized by ion trapping.

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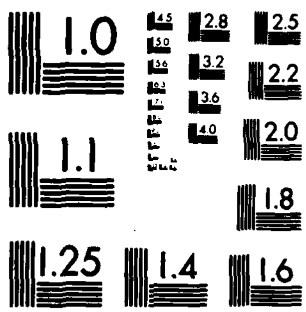
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INTRODUCTION

If a laser beam is injected into plasma, the energy of the laser field is transferred to the electrons by the following mechanisms:

(i) In part, the laser radiation is transformed into plasma waves through interactions with electrons or the already existing plasma waves. Then, energy flows from plasma waves to electrons by Landau damping, which diffuses the electrons in velocity space. This mechanism is known not only as the only one heating mechanism caused by the plasma waves but also as the responsible mechanism for the anomalously large heating of collisionless plasmas by lasers [1]. Hence, this mechanism has been called either turbulent heating or anomalous heating by other authors [1,2,3]. However, we call it "classical turbulent heating" since this mechanism turnes out to be neither the sole heating mechanism of the plasma waves nor the dominant cause for the so-called anomalous heating.

(ii) The direct absorption of the laser radiation by the individual electrons under the influence of the plasma waves (collisionless inverse bremsstrahlung) [4].

(iii) The same process as collisionless inverse bremsstrahlung, but under the influence of the static plasma field (collisional inverse bremsstrahlung) [5].

These three mechanisms are independent but can occur concomitantly as one readily sees from the quantum-mechanically extended Vlasov equation (Kim - Wilhelm equation).

The first systematic theory of the dissipation of an electromagnetic wave in plasmas has been given by Dawson and Oberman [6]. However, since their theory is based on the hydrodynamic momentum equation (force equation) of the electrons only, the heating of the electrons cannot be treated by their theory in principle (only the over-all ohmic dissipation of the electromagnetic wave energy, including scattering, can be determined). Furthermore, the application of their theory is limited in the weak laser-intensity regime where anomalous heating by laser-induced plasma waves does not exist.

Recently, Manheimer [7] has conjectured an equation for the heating of the electrons in the laser-irradiated plasma by adding a phenomenological term for the anomalous heating to the energy equation of the electrons [7,8]. The latter appears to have been guessed from the Vlasov equation without taking in account properly the laser field. The term for the so-called anomalous heating, which is supposed to consider heating by collisionless inverse bremsstrahlung, is purely phenomenological, i.e., is not derived from first principles.

For these reasons, herein the equation for the heating rate of the electrons of the plasma in the field of high-frequency electromagnetic waves is derived from the quantum-mechanically extended Vlasov equation. The quantum-mechanically extended Vlasov equation is obtained from the ordinary classical Vlasov equation by replacing the term for the change of the electron distribution due to acceleration (or deceleration) of the electrons by the laser radiation through a term representing the classical limit of the change in the electron distribution due to absorption (or emission) of laser quanta [9]. The equation for the heating rate of electrons here gives the detailed features as to how and what amount of the energy of the laser radiation is transferred to the electrons. Especially the dominant mechanism for the anomalous heating of collisionless plasmas by lasers is clearly revealed by this equation.

BASIC EQUATIONS

We begin by adopting the set of Vlasov equations to describe the change of the electron distribution in the $(\vec{r},\vec{v})$ space, and the time-dependent Schroedinger equation to describe the dynamics of the individual electrons in the $\vec{r}$-space. The ions are regarded as a set of randomly distributed fixed scatters.

We assume that the laser light is a circularly polarized electromagnetic wave propagating in the z-direction. The spatial dependence of this electromagnetic wave is neglected since the wave length of the electromagnetic wave is assumed to be much longer than that of the plasma waves. Under these assumptions, the field equations of the plasma are (Gaussian units):

$$\frac{\partial f}{\partial t} + \vec{v}\cdot \frac{\partial f}{\partial \vec{r}} - \frac{e}{m}(\vec{E}_{e.m.} - \nabla\phi)\cdot\frac{\partial f}{\partial \vec{v}} = 0 \quad , \quad (1)$$

$$\nabla^2\phi = 4\pi[e\int f\, d^3v - Ze\sum_j \delta(\vec{r} - \vec{r}_j)] \quad , \quad (2)$$

$$i\hbar\frac{\partial \Psi}{\partial t} = [(1/2m)(\frac{\hbar}{i}\nabla + \frac{e}{c}\vec{A})^2 - e\phi]\Psi \quad , \quad (3)$$

where

$$\vec{E}_{e.m.} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} \quad , \quad (4)$$

$$\vec{A}(t) = A_o(\hat{x}\cos\omega t + \hat{y}\sin\omega t) \quad . \quad (5)$$

$\phi(\vec{r},t)$ is the potential of the plasma field (the plasma waves plus the static plasma field). The subscript e.m. is an abbreviation for "electromagnetic", and the other symbols have the usual meaning.

The potential of the plasma field can be expanded in a double Fourier series,

$$\phi(\vec{r},t) = \sum_{\vec{k}} \sum_{\Omega} \phi(\vec{k},\Omega) e^{i(\vec{k}\cdot\vec{r}-\Omega t)} \quad . \tag{6}$$

(For the case that the static plasma field is negligible, $\phi(\vec{r},t)$ can be expanded in a single Fourier series since in this case $\vec{k}$ and Ω are interrelated by the dispersion relation). Since $\phi(\vec{r},t)$ is real, we have $\phi(-\vec{k},-\Omega) = \phi(\vec{k},\Omega)^*$.

After substituting Eqs. (5) and (6) into Eq. (3), we solve for the transition probability amplitude from the resulting equation by means of first-order perturbation theory [4,5] where ϕ is the pertubing potential. The transition probability amplitude from a state 1 with momentum $\vec{p}_1$ to a state 2 with momentum $\vec{p}_2$ of the electron is [4,5]

$$a(1 \rightarrow 2) = -\frac{2ie}{\hbar} e^{-ic'} \sum_{\vec{k}} \sum_{\Omega} \phi(-\vec{k},-\Omega) \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{in\delta} J_n(\lambda/\omega) \frac{\sin[(\frac{\varepsilon}{\hbar} - \Omega - n\omega)T]}{(\varepsilon/\hbar) - \Omega - n\omega} \quad , \tag{7}$$

where

$$\lambda = eE_o k_\perp/m\omega \quad , \quad \varepsilon = (p_2^2 - p_1^2)/2m \quad . \tag{8}$$

δ is the azimuthal angle of $\vec{k}$, i.e., $\vec{k} = (k_\perp,\delta,k_z)$, and c' is a real constant, which drops out since exp(-ic') does not affect the transition probability. The subscript $\perp$ refers to the direction perpendicular to the direction of propagation of the electromagnetic wave, and E_o is the electric field amplitude of the electromagnetic wave.

From Eq. (7) we see that the transition probability per unit time for the transition from state 1 to state 2 (with the absorption or the emission of photons) is

$$T(1 \to 2) = T(\vec{v}_1, \vec{v}_2) = |a(1 \to 2)|^2/2T$$

$$= \frac{2\pi e^2}{\hbar} \sum_{\vec{k}} \sum_{\Omega} |\phi(\vec{k},\Omega)|^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} J_n^2(\lambda/\omega)\delta(\varepsilon - \hbar\Omega - n\hbar\omega) \quad . \qquad (9)$$

In the derivation of Eq. (9), we assumed that there are no phase relations between different components of the plasma field.

The change of the electron distribution due to the absorption or emission of photons of the electromagnetic wave [4,5] is

$$[\partial f(\vec{v}_2)/\partial t]_{e.m.} = \sum_1 [T(1 \to 2)f(\vec{v}_1) - T(2 \to 1)f(\vec{v}_2)] = \sum_1 T(\vec{v}_1,\vec{v}_2)[f(\vec{v}_1) - f(\vec{v}_2)] \qquad (10)$$

where

$$\vec{v}_1 = \vec{p}_1/m = 2\pi\hbar m^{-1}(n_x/L_x, n_y/L_y, n_z/L_z) \quad , \quad n_x, n_y, n_z = \text{integers},$$

is quantized (L_x, L_y, L_z are the x-,y-,z-dimensions of the plasma, respectively).

In Eq. (1), $(e/m)\vec{E}_{e.m.} \cdot \partial f/\partial \vec{v}$ is the rate of change in the number of electrons per unit volume in the $(\vec{r},\vec{v})$space due to acceleration (or deceleration) of electrons by the laser field. From the quantum-mechanical viewpoint, the acceleration of an electron by the spatially uniform laser field is possible only by absorption (or emission) of the laser photons in the presence of another field which is the plasma field in our situation. Hence, $(e/m)\vec{E}_{e.m} \cdot \partial f/\partial \vec{v}$ is equal to the classical limit $(h \to 0)$ of the rate of change in the mumber of electrons per unit volume in the $(\vec{r},\vec{v})$ space due to the absorption (or emission) of the laser photons by the electrons under the influence of the plasma field ("inverse bremsstrahlung"). Accordingly, by using Eqs. (1), (3) - (6), and the same methods as in Ref. 5 and 6, we obtain the quantum-mechanical Vlasov equation, which is valid up to second order in ϕ:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot \frac{\partial f}{\partial \vec{r}} + \frac{e}{m}\nabla\phi\cdot\frac{\partial f}{\partial \vec{v}} = \left(\frac{\partial f}{\partial t}\right)_{i.b.} \quad , \qquad (11)$$

where

$$\left[\frac{\partial f(v)}{\partial t}\right]_{i.b.} = \lim_{\hbar\to 0}\sum_{\vec{v}'} T(\vec{v}' \to \vec{v})[f(\vec{v}') - f(\vec{v})]$$

$$= \frac{\pi e^2}{m^2}\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} J_n^2\left(\frac{\lambda}{\omega}\right)\left[\frac{\partial\delta\left(v_{\vec{k}} - \frac{\Omega+n\omega}{k}\right)}{\partial v_{\vec{k}}}\frac{\partial f(\vec{v})}{\partial v_{\vec{k}}} + \delta\left(v_{\vec{k}} - \frac{\Omega+n\omega}{k}\right)\frac{\partial^2 f(\vec{v})}{\partial v_{\vec{k}}^2}\right], \qquad (12)$$

with $v_{\vec{k}} = \vec{v}\cdot\vec{k}/k$. The subscript i.b. is an abbreviation for "inverse bremsstrahlung". In the derivation of Eq. (12), $v_{-\vec{k}} = -v_{\vec{k}}$, and $\phi(-\vec{k},-\Omega) = \phi(\vec{k},\Omega)^*$ have been used to cancel out the terms, which change their signs as $(\vec{k},\Omega,n)$ changes to $(-\vec{k},-\Omega,-n)$, in the summation over $\vec{k},\Omega$, and n.

Equation (11) is the desired Vlasov equation of the electrons in the electromagnetic field and the plasma field, in which the term $(\partial f/\partial t)_{i.b.}$ considers all interactions of the electrons with electromagnetic field. It should be noted that the presented derivation of Eq. (11) is based on first principles (Vlasov and Schroedinger equations in electromagnetic and plasma fields).

Since Eq. (11) is valid up to the second order in ϕ, we can apply to this equation an analogy to the derivation of the quasilinear equation for the electron distribution in the absence of the electromagnetic wave [10,11]. The result, i.e., the quasilinear equation for changes in the electron distribution in the presence of the electromagnetic wave, is by Eq. (11)

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{c.t.} + \left(\frac{\partial f}{\partial t}\right)_{i.b.} \quad , \qquad (13)$$

where

$$\left(\frac{\partial f}{\partial t}\right)_{c.t.} = -i\frac{e^2}{m^2}\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2 \frac{\partial}{\partial\vec{v}}\cdot\left[\frac{\vec{k}(\vec{k}\cdot\partial f/\partial\vec{v})}{\vec{k}\cdot\vec{v} - \Omega - i\gamma}\right] \qquad (14)$$

is the change in the electron distribution associated with the diffusion in the electron-velocity space due to the acceleration of the electrons by the electric field of the plasma waves (or the change in the electron distribution by classical turbulent heating). The subscript c.t. is an abbreviation for "classical turbulent heating", and $\gamma > 0$ is infinitesimal. In Eqs. (13) and (14), the spatial average of the actual distribution function is designated as f, and the same simplification in notation is adopted in the following, where there is no chance of confusion.

HEATING RATE OF ELECTRONS IN COLLISIONLESS PLASMA

The heating rate of the electrons per unit volume under the influence of the external electromagnetic and internal plasma fields is by microscopic definition

$$\frac{\partial\langle\varepsilon\rangle}{\partial t} = \int_{-\infty}^{+\infty} d^3v \frac{1}{2} Nm(\vec{v} - \vec{u})^2 [(\frac{\partial f}{\partial t})_{i.b.} + (\frac{\partial f}{\partial t})_{c.t.}] = (\frac{\partial\langle\varepsilon\rangle}{\partial t})_{i.b.} + (\frac{\partial\langle\varepsilon\rangle}{\partial t})_{c.t.} \qquad (15)$$

where N is the unperturbed electron density. In Eq. (15), the first term is the heating rate of the electrons per unit volume by inverse bremsstrahlung, and the second term is that by classical turbulent heating, i.e., the rate of energy transfer to the electrons from the plasma waves by velocity-space diffusion.

We model the spatial average of the electron distribution as

$$f(\vec{v}) = (2\pi v_e^2)^{-3/2} \exp[-\frac{(\vec{v}-\vec{u})^2}{2v_e^2}]\left\{1 + \frac{[(\vec{v}-\vec{u})\cdot\vec{Q}]^2}{3Nmv_e^4}\left(\frac{[(\vec{v}-\vec{u})\cdot\vec{Q}]^2}{v_e^2 Q^2} - 3\right)\right\} \qquad (16)$$

where the hydrodynamic velocity $\vec{u}$ and heat flux $\vec{Q}$ are induced by the laser field.

In the appendix we calculate the second term of Eq. (15) as

$$(\frac{\partial\langle\varepsilon\rangle}{\partial t})_{c.t.} = (\frac{\pi}{2})^{1/2} \frac{Ne^2}{mv_e^2} \sum_{\vec{k}} \sum_{\Omega} \frac{|\phi(\vec{k},\Omega)|^2}{k} \exp[-\frac{(\Omega-\vec{k}\cdot\vec{u})^2}{2v_e^2k^2}](\Omega - \vec{k}\cdot\vec{u})$$

$$\times\left\{\Omega - \vec{k}\cdot\vec{u} + \frac{(\vec{k}\cdot\vec{Q})^3}{Nmv_e^2k^2Q^2}[1 - 2(\frac{\Omega-\vec{k}\cdot\vec{u}}{kv_e})^2 + \frac{1}{3}(\frac{\Omega - \vec{k}\cdot\vec{u}}{kv_e})^4]\right\}. \qquad (17)$$

The calculation of the first term by means of Eqs. (12), (15), and (16) is elementary, and gives

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{i.b.} = -\frac{\pi e^2}{m^2}\sum_{\vec{k}}\sum_{\Omega} k|\phi(\vec{k},\Omega)|^2 \int \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} J_n^2\left(\frac{\lambda}{\omega}\right)\delta\left(v_{\vec{k}} - \frac{\Omega+n\omega}{k}\right)\frac{\partial f}{\partial v_{\vec{k}}}\frac{\partial}{\partial v_{\vec{k}}}\left[\frac{1}{2}Nm(\vec{v}-\vec{u})^2\right]d^3v$$

$$= \left(\frac{\pi}{2}\right)^{1/2}\frac{Ne^2}{mv_e^3}\sum_{\vec{k}}\sum_{\Omega}\frac{|\phi(\vec{k},\Omega)|^2}{k}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} J_n^2\left(\frac{\lambda}{\omega}\right)\exp\left[-\frac{(\Omega+n\omega-\vec{k}\cdot\vec{u})^2}{2v_e^2k^2}\right](\Omega+n\omega-\vec{k}\cdot\vec{u})$$

$$\times\left\{\Omega + n\omega - \vec{k}\cdot\vec{u} + \frac{(\vec{k}\cdot\vec{Q})^3}{Nmv_e^2k^2Q^2}\left[1 - 2\left(\frac{\Omega+n\omega-\vec{k}\cdot\vec{u}}{v_ek}\right)^2 + \frac{1}{3}\left(\frac{\Omega+n\omega-\vec{k}\cdot\vec{u}}{v_ek}\right)^4\right]\right\} \quad . \qquad (18)$$

Equations (17) and (18) can be used irrespective of the origin of the plasma field as long as the strength of this field is so small that the resulting perturbation of the electron distribution can be adequately treated by means of first-order perturbation theory. Bearing this in mind, we reduce the expression of the heating rate to a simpler form for the cases of small and large λ/ω.

(i) $\lambda/\omega \ll 1$: In this case, the laser-induced current and heat flux can be neglected. Then, by means of the same approach used to derive Eqs. (1) - (15) of Ref. 6, we ontain from Eqs. (1) and (2)

$$\phi(\vec{r},t) = \frac{(2\pi)^3}{V}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}[S(\vec{k}) - C(\vec{k})^* e^{-i\delta}e^{i\omega t} + C(\vec{k})e^{i\delta}e^{-i\omega t}] \quad , \qquad (19)$$

where

$$S(\vec{k}) = \frac{Ze}{2\pi^2(k^2+k_D^2)}\sum_j e^{-i\vec{k}\cdot\vec{r}_j} \quad , \qquad (20)$$

$$C(\vec{k}) = \frac{Ze}{4\pi^2}\frac{\lambda}{\omega}\left(\sum_j e^{-i\vec{k}\cdot\vec{r}_j}\right)k^{-2}\left[\frac{1}{D(k,0)} - \frac{1}{D(k,\omega)}\right]^* \qquad (21)$$

with

$$D(k,\omega) = 1 + \frac{k_D^2}{k^2}W(s) \quad , \quad s = -\frac{\omega}{v_ek} \quad . \qquad (22)$$

Here, $D(k,\omega)$ is the longitudinal dielectric constant, V is the volume of the plasma system, $k_D = \omega_p/v_e$ is the Debye wave number, ω_p the plasma frequency, and W(s) is the plasma function given in the appendix.

Designating by $\phi_j(\vec{r})$ the Coulomb field potential at $\vec{r}$ due to an ion at $\vec{r}_j$, and by λ_D the Debye length, the first term of Eq. (19) becomes

$$\phi(\vec{r}) = \sum_j \phi_j(\vec{r}) = \sum_j \frac{Ze}{|\vec{r}-\vec{r}_j|} \exp(-|\vec{r}-\vec{r}_j|/\lambda_D) \quad , \qquad (23)$$

Hence, the first term of Eq. (19) is the static Coulomb field potential, reduced by the usual Debye shielding, acting on an electron due to all ions in the plasma. This static plasma field gives rise to collisional inverse bremsstrahlung but does not contribute to classical turbulent heating. The second and third terms are the potential of the laser-induced plasma waves, which can contribute both to collisionless inverse brensstrahlung and classical turbulent heating.

From Eqs. (18) - (21) and

$$J_n^2(\frac{\lambda}{\omega}) = \frac{1}{(n!)^2}(\frac{\lambda}{2\omega})^{2|n|} \qquad \text{for } \lambda/\omega << 1 \quad , \qquad (24)$$

it is seen that the collisionless inverse bremsstrahlung is of the fourth order in λ/ω, and can be neglected compared with the collisional inverse bremsstrahlung, which is of the second order, Hence, up to the lowest order in λ/ω, we obtain from Eqs. (18) - (21), and (24),

$$(\frac{\partial<\varepsilon>}{\partial t})_{i.b.} = (2\pi)^{7/2} \frac{Ne^2}{mv_e^3}\int \frac{d^3k}{k}\frac{1}{4}\frac{|S(\vec{k})|^2}{V}(\frac{\lambda}{\omega})^2\omega^2 \exp(-\omega^2/2v_e^2k^2) \quad . \qquad (25)$$

The expression for $\phi_j(\vec{r})$ given in Eq. (23) shows that $\phi_j(\vec{r})$ decreases very fast to zero as $|\vec{r}-\vec{r}_j|/\lambda_D \to \infty$. This decrease is mathematically brought about by representing discrete phenomena through smoothly continuous functions; $\phi_j(\vec{r})$ is in reality zero for $|\vec{r}-\vec{r}_j| >> \lambda_D$. This corresponds to $S(\vec{k}) = 0$ for $k << k_D$. Considering this behavior of $S(\vec{k})$ and Eq. (20), we can explicitly evaluate Eq. (25) as

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{i.b.} \cong \frac{4}{3}(2\pi)^{1/2}\frac{Z^2e^6E_o^2N^2}{m^3v_e^3\omega^2}\int_0^{\sim k_D}\frac{dk\ k^3\exp(-\omega^2/2v_e^2k^2)}{(k^2+k_D^2)^2} = \frac{4}{3}(2\pi)^{1/2}\frac{Z^2e^6E_o^2N^2}{m^3v_e^3\omega^2}U(z) , \tag{26}$$

Where

$$U(z) = \int_0^z dx[x^3\exp(-1/2x^2)/(x^2+z^2)^2] , \qquad z = \omega_p/\omega , \tag{27}$$

with ω_p the plasma frequency. The function U(z) is represented in Fig. 1.

In the derivation of Eq. (26), we assumed the ion distribution to be random so that

$$\frac{1}{V}\left|\sum_j \exp(-i\vec{k}\cdot\vec{r}_j)\right|^2 = N \tag{28}$$

since the large-amplitude ion wave cannot be induced by weak laser fields for which $\lambda/\omega << 1$ [12].

From Eqs. (17) and (19), we obtain for the heating rate per unit volume by classical turbulent heating, up to the lowest order in λ/ω,

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{c.t.} \cong (2\pi)^{7/2}\frac{e^2N}{mv_e^3}\int\frac{d^3k}{k}\frac{|C(\vec{k})|^2}{V}\omega^2\exp(-\omega^2/2v_e^2k^2) . \tag{29}$$

Eq. (29) can be expressed more explicitly for the cases of hot and cold plasmas. In this connection, it is noted that electromagnetic waves propagate

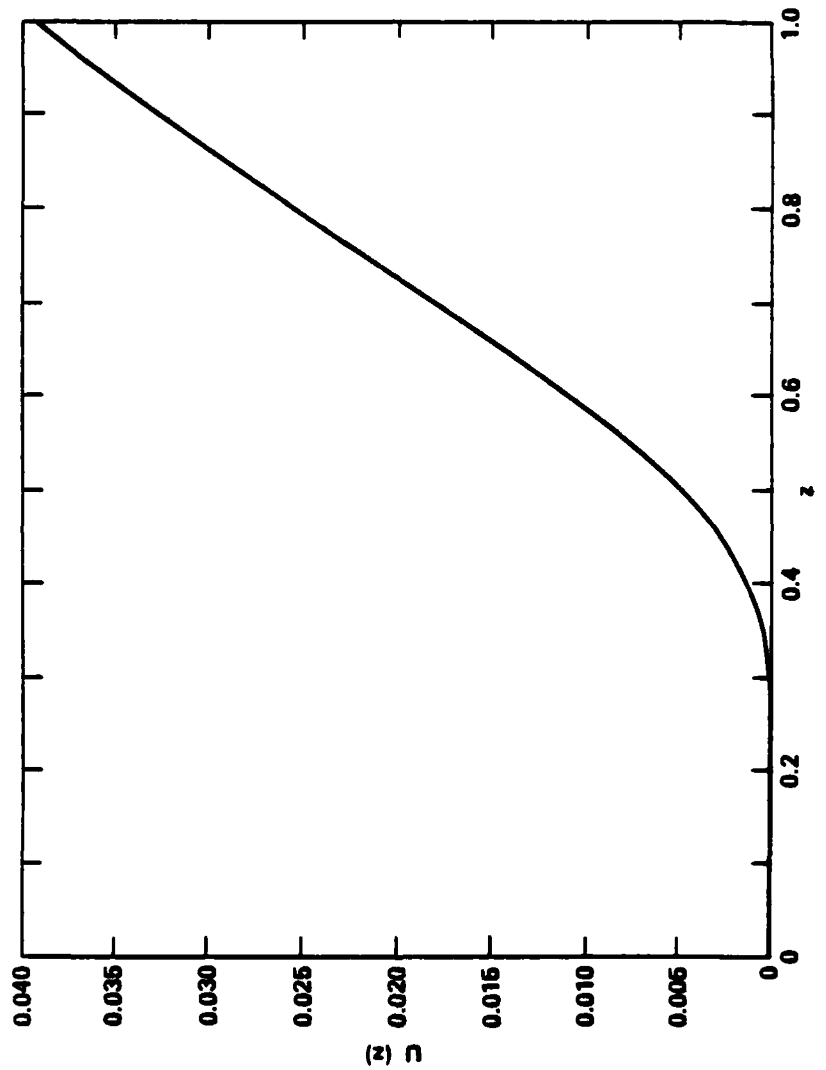

Fig. 1: U(z) versus z by Eq. (27)

in plasmas only when their frequency is greater than the plasma frequency. Accordingly,

$$v_e < \omega v_e/\omega_p = \omega/k_D \quad . \tag{30}$$

For hot plasmas for which $v_e >> \omega/k$, Eq. (22) becomes, to the lowest order in ω/kv_e, such that $[1/D(k,0) - 1/D(k,\omega)]$ is not zero,

$$D(k,\omega) = 1 + k_D^2/k^2 - i(\pi/2)^{1/2}(k_D^2/k^2)\frac{\omega}{v_e k} \quad . \tag{31}$$

Also, from $v_e >> \omega/k$ and Eq. (30), we have $k >> k_D$. This means that the plasma behaves, as far as the plasma waves are concerned, like a system of free individual particles [13].

Substituting Eqs. (21) and (31) into Eq. (29) and using $k >> k_D$ yields

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{c.t.} = \frac{1}{3}(2\pi)^{1/2}\frac{Z^2e^6E_o^2N^2}{m^3v_e^3\omega_p^2}\int_{-1}^{\infty}\frac{dx\ x}{(1+x^2)^4} \simeq \frac{(2\pi)^{3/2}Z^2e^6E_o^2N^2}{144m^3v_e^3\omega_p^2} \quad . \tag{32}$$

By comparison of Eq. (32) with Eq. (26), it is recongized that the heating rate of the electrons by inverse bremsstrahlung is slightly greater than that by classical turbulent heating for $\omega = \omega_p$.

For cold plasmas for which $v_e < \omega/k_D << \omega/k$, Eq. (22) becomes, to the lowest order in $v_e k/\omega$ such that $[1/D(k,0) - 1/D(k,\omega)]$ in Eq. (21) is not zero,

$$D(k,\omega) = 1 - \omega_p^2/\omega^2 \quad . \tag{33}$$

Accordingly, Eq. (29) reduces to

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{c.t.} \simeq \frac{4}{3}(2\pi)^{1/2}\frac{Z^2e^6E_o^2N^2}{m^3v_e^3\omega^2}\frac{\omega_p^4}{(\omega^2-\omega_p^2)^2}V(z) \quad , \tag{34}$$

where

$$V(z) = \int_0^z dx[\exp(-1/2x^2)/x(x^2+z^2)^2] \quad . \quad (35)$$

The function V(z) is represented in Fig. 2.

Equation (34) appears to indicate that classical turbulent heating by the plasma wave becomes resonant as $\omega \to \omega_p$. However, Eq. (34) should be questioned since Eq. (38) is derived by using Eqs. (19) and (21), which are derived from the collisionless Boltzmann equation (or Vlasov equation), even though the considered plasmas are cold, and consequently, collisional.

Comparison of the expression for the heating rate by inverse bremsstrahlung, i.e., Eq. (26) with that found by Seely and Harris [Eq. (15) of Ref. 5] shows that our result is the same as theirs except for a factor $U(z)/\ln\Lambda$, where $\ln\Lambda$ is the Coulomb logarithm,

$$\ln\Lambda = \ln(12\pi N\lambda_D^3) \quad . \quad (36)$$

Since the Coulomb logarithm for laboratory plasmas is about 10 to 20 and U(z) is less than 0.04, the value of the heating rate given by Eq. (26) is less than about four-thousandth to two-thousandth of that given by Eq. (15) of Ref. 5. This difference is due to the fact that Seely and Harris do not consider Debye shielding, whereas our result contains Debye shielding brought about by the collective dynamics of the electrons [Eq. (19)].

(ii) $\lambda/\omega = eE_o k_1/m\omega^2 >> 1$: For this case, we can use no longer Eqs. (19) - (21) for the plasma field potential ϕ even if there is no other perturbation except the electromagnetic field. However, we can readily surmise that the collision<u>less</u> inverse bremsstrahlung is stronger than the collisional one for $\mu \equiv \lambda/\omega >> 1$. This is because the second and third terms of Eq. (19), which

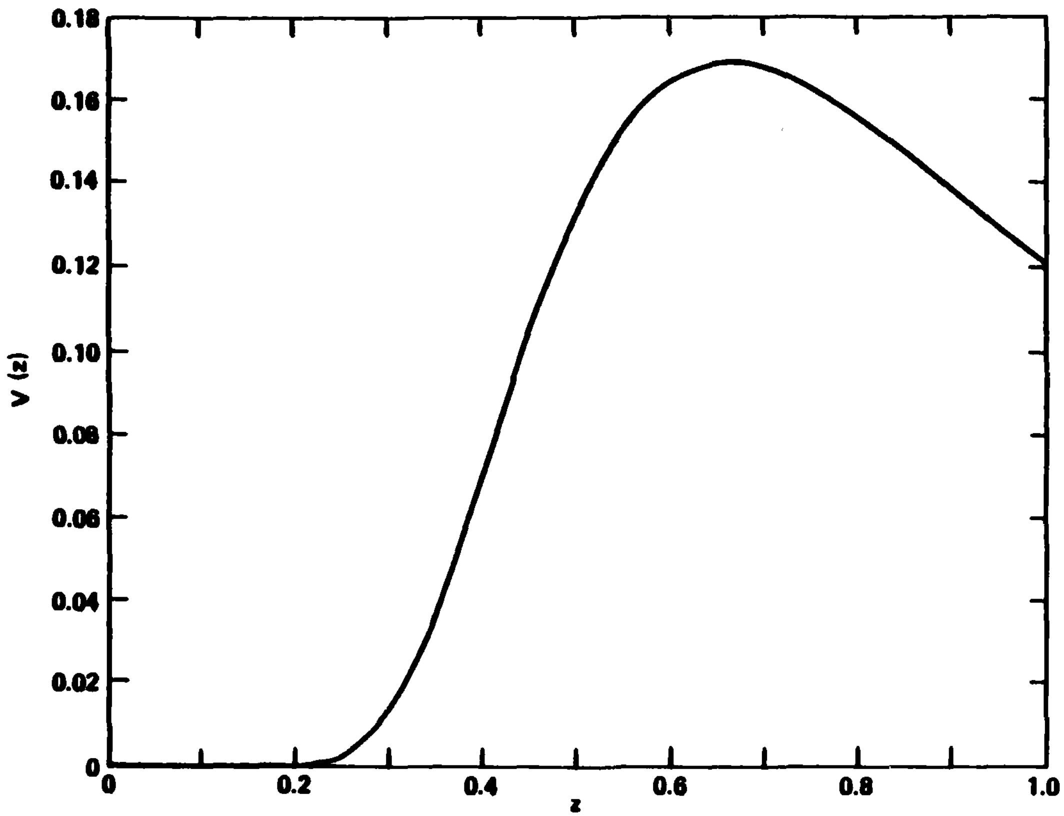

Fig. 2: V(z) versus z by Eq. (35)

give rise to collisionless inverse bremsstrahlung, become important compared to the first term, which produces collisional inverse bremsstrahlung, as μ increases (but still $\mu < 1$). For this reason, the result obtained by Seely and Harris for the effective collision frequency in the case $\mu >> 1$ (Eq. (22) of Ref. 5] is questionable since they take into account only collisional inverse bremsstrahlung.

Except for increasing μ (usually by increasing the intensity of the electromagnetic field), the collisionless inverse bremsstrahlung becomes more important compared with the collisional one with increasing electron temperature. This is because the mean free path of the electron becomes larger with increasing electron energy. Accordingly, the collective effects which are enhanced by a large number of instabilities (not caused by the local electromagnetic field) become more dominant than the effect caused by the static ions. Hence, even if $\mu < 1$, the collisionless inverse bremsstrahlung may dominate the collisional one for very hot plasmas if there are causes of turbulence other than the local electromagnetic field. The latter case will be treated in the next section.

For $\mu = \lambda/\omega >> 1$, we can write approximately [5]

$$\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} J_n^2\left(\frac{\lambda}{\omega}\right)\delta\left(v_{\vec{k}} - \frac{\lambda + n\omega}{k}\right) = \frac{1}{2}\left[\delta\left(v_{\vec{k}} - \frac{\Omega - \lambda}{k}\right) + \delta\left(v_{\vec{k}} - \frac{\Omega + \lambda}{k}\right)\right] \quad . \tag{37}$$

Substituting Eq. (37) into the first relation of Eq. (18), we find for hot plasmas, $v_e >> \Omega/k,\ \lambda/k$,

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{i.b.} \doteq \left(\frac{\pi}{2}\right)^{1/2} \frac{Ne^2}{mv_e^3} \sum_{\vec{k}} \sum_{\Omega} \frac{|\phi(\vec{k},\Omega)|^2}{k} \left\{(\Omega - \vec{k}\cdot\vec{u})\left[\Omega - \vec{k}\cdot\vec{u} + \frac{(\vec{k}\cdot\vec{Q})^3}{Nmv_e^2k^2Q^2}\right] + \frac{e^2E_o^2k_\perp^2}{m^2\omega^2}\right\} . \tag{38}$$

Comparison of Eq. (38) with Eq. (17) indicates that the heating of the electrons by collisionless inverse bremsstrahlung is greater than that by classical turbulent heating where the following conditions hold: $\mu >> 1$ and $v_e >> \Omega/k$, λ/k (hot plasmas).

ANOMALOUS HEATING RATE

Heating by collisionless inverse bremsstrahlung, which is the main heating mechanism in laser fusion, is usually called "anomalous" heating. This phenomenon requires a correct quantum-mechanical treatment of collisionless inverse bremsstrahlung, as distinguished from the deficient classical approaches in Refs. [6,7, 8,12,13]. From the experimental point of view, the heating rate is "anomalous" in collisionless plasmas since it is unexpectively large compared with the classical theories, which consider collision-dominated phenomena. In the following, the significance of collisionless inverse bremsstrahlung is demonstrated by comparison with the classical theory.

Equation (38) is valid irrespective of the causes of the electrostatic field. In laser fusion, it is believed that the electrostatic field is developed by the ion acoustic instability driven by the return current [7]. For this case, it is known from computer simulations that the $\vec{k}$-spectrum of the electrostatic field is flared out in a cone of 45° to the direction of the return current [7,15-17]. Hence, we can assume that $k_\perp$ is of the order of k. Then, the condition $\mu > 1$ corresponds to a laser intensity of $I > 10^{12}$ watt/cm^2 for a neodymium laser. With presently available laser powers, this condition is generally satisfied in most experiments so that Eq. (38) is applicable to them.

Both computer simulations and analytical theories indicate that the ion acoustic instability is "stabilized" by ion trapping [18-22]. Hence, its intensity level can be estimated on the basis of simple trapping arguments to be [7]

$$|\phi(k,\Omega)| \simeq \frac{KT_e}{4e}\left[\frac{k_D}{(k^2+k_D^2)^{1/2}} - \frac{3T_i}{T_e}\right]^2 \qquad (39)$$

where the wave spectrum of the turbulent electric field is modeled for a single wave having wave number k, frequency Ω, and a propagation angle of 135° with the direction of the laser beam. By incorporating the estimate (39) for the saturated turbulence level into Eq. (38), we obtain the anomalous heating rate for hot plasmas with $v_e >> \lambda/k$:

$$\left(\frac{\partial<\varepsilon>}{\partial t}\right)_{anom.} = 0.02\frac{e^2NE_o^2kv_e}{m\omega^2}\left[\frac{k_D}{(k^2 - k_D^2)^{1/2}} - \frac{3T_i}{T_e}\right]^4 \quad . \tag{40}$$

By Eqs. (26), and (40), the ratio R of the anomalous heating rate to the heating rate by the collisional inverse bremsstrahlung in very hot (collisionless) plasmas is, assuming $T_e >> T_i$ and $k \simeq k_D/2$:

$$R \simeq N\lambda_D^3/U(z) \geq 6N_D \quad , \quad N_D = 4\pi\lambda_D^3N/3 \quad , \tag{41}$$

where N_D is the number of electrons in a Debye sphere [23]. By definition, in "collisionless plasmas" the collective behavior far dominates the collisional effects [23], and $N_D >>> 1$. Thus, by Eq. (41) we demonstrate the significance of our collisionless quantum-mechanical theory compared with the classical collisional theories and explain the observed anomalous heating rate in experiments, $R \simeq 6N_D >>> 1$.

CONCLUSION

Under the condition that the field strength of the electromagnetic wave is below the threshold above which strong turbulence is developed (so that the first-order perturbation theory would no longer be appropriate) we have obtained the following results:

(1) From the set of Vlasov and Schroedinger equations, the heating rate of the electrons in the electromagnetic field is systematically derived in terms of the electrostatic field potential, which is produced by the static ions and the collective dynamics of electrons in the self-consistent manner. The Vlasov equation is generalized by formulating in it $\frac{e}{m}\vec{E}_{e.m.} \cdot \partial f/\partial \vec{v}$ as a quantum-mechanical interaction integral.

(2) Especially for $\mu < 1$, the electrostatic field potential ϕ in the heating rates is expressed in terms of the field strength E_o, frequency ω , and wave vector $\vec{k}$ of the electromagnetic wave for both hot and cold plasmas.

(3) For very hot plasmas with $v_e >> \Omega/k, \lambda/k$, it is shown that the heating of the electrons by inverse bremsstrahlung is considerably stronger than that by Landau damping of electrostatic waves. Furthermore, it is shown that for $\mu < 1$ the above statement is true regardless of the magnitude of the electron temperature.

(4) The anomalous heating observed in the experiments is identified as the heating by inverse collisionless bremsstrahlung, and explicitly formulated when the turbulence is developed by the ion acoustic instability and "stabilized" by ion trapping.

APPENDIX

Combination of Eqs. (14) - (16) yields

$$\left(\frac{\partial\langle\varepsilon\rangle}{\partial t}\right)_{c.t.} = -iN \frac{e^2}{mv_e} \sum_{\vec{k}} \sum_{\Omega} k|\phi(\vec{k},\Omega)|^2 (2\pi)^{-1/2}$$

$$\times \int_{-\infty}^{+\infty} \frac{dx \exp(-x^2/2)}{x - z - i\gamma} \left[x^2 + \frac{1}{3Nmv_e^3 Q^2} \left(\frac{\vec{k}\cdot\vec{Q}}{k}\right)^3 (x^5 - 6x^3 + 3x)\right] \qquad . \qquad (A1)$$

Integration over x in Eq. (A1) is accomplished by using the following formulae:

$$(2\pi)^{-1/2} \int_{-\infty}^{+\infty} \frac{dx\ x \exp(-x^2/2)}{x - z - i\gamma} = W(z) \qquad , \qquad (A2)$$

$$(2\pi)^{-1/2} \int_{-\infty}^{+\infty} \frac{dx\ x^2 \exp(-x^2/2)}{x - z - i\gamma} = zW(z) \qquad , \qquad (A3)$$

$$(2\pi)^{-1/2} \int_{-\infty}^{+\infty} \frac{dx\ x^3 \exp(-x^2/2)}{x - z - i\gamma} = 1 + z^2 W(z) \qquad , \qquad (A4)$$

$$(2\pi)^{-1/2} \int_{-\infty}^{+\infty} \frac{dx\ x^5 \exp(-x^2/2)}{x - z - i\gamma} = 3 + z^2[1 + z^2 W(z)] \qquad , \qquad (A5)$$

where W(z) is the plasma function [24] given by

$$W(z) = \begin{cases} i\left(\frac{\pi}{2}\right)^{1/2} z \exp\left(-\frac{z^2}{2}\right) + \left[1 - z^2 + \frac{z^4}{3} - \cdots + \frac{(-1)^{n+1} z^{n+2}}{(2n+1)!!} + \cdots\right], \text{for } |z| < 1; \\ i\left(\frac{\pi}{2}\right)^{1/2} z \exp\left(-\frac{z^2}{2}\right) - \left[\frac{1}{z^2} + \frac{3}{z^4} + \cdots + \frac{(2n-1)!!}{z^{2n}} + \cdots\right], \text{ for } |z| > 1. \end{cases} \qquad (A6)$$

Substitution of Eqs. (A2) - (A6) into Eq. (A1) and obliteration of all odd terms for both $\vec{k}$ and Ω in the summation over $\vec{k}$ and Ω gives Eq. (17).

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Detonation driven magnetogasdynamic generators with external R-L load circuits are analyzed, for which the external and induced magnetic fields are parallel and antiparallel, respectively. Two plasma shock flow models are treated: (i) implosion produced jet flow and (ii) plane detonation flow. — A theory of power generation is developed for a stress polarizable solid between plane electrodes (with an external ohmic load circuit), which is electrically polarized by an explosion produced stress shock wave. — In connection with the analysis of flux compression power generators, initial-boundary-value problems for the diffusion of magnetic fields into conductors with external electromagnetic transients and the electromagnetic induction in conductors accelerated in external magnetic fields are solved. — A quantum-kinetic theory of the anomalous electron heating in plasmas by high-frequency electromagnetic fields is presented.

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